Endogenous Growth and European Fiscal Rules

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Abstract

The creation of the European Monetary Union has led to a substantial increase in the discussion of the importance of fiscal discipline and adequate fiscal rules in such a monetary union. The “European” solution has been challenged by many authors and politicians: among the main questions discussed in recent years, we find the use of the same rules for different situations in Member-Countries, particularly in terms of economic dimension and economic level of development.

We develop a model of a monetary union between two countries that may differ in economic dimension and in the level of development. By solving transitional dynamics towards the steady state through numerical computation, the model allows us to examine the impact of fiscal shocks that may lead to excessive deficits.

Our results suggest that the implications of such deficits depend on whether they occur in the small and less developed country or in the big and more developed one. In this context, we argue that an excessive deficit should be temporarily allowed in the case of the small and less developed country, in order to improve economic convergence and wages within the union.

Keywords: Monetary Union; SGP; Excessive Deficits; Technological-Knowledge Gap; Numerical Computation.

JEL Classification: C61, E62, H6, O3

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1. Introduction

With the creation of the European Monetary Union (EMU), the framework for the definition and implementation of macroeconomic policies has dramatically changed. Member-Countries have lost their exchange rate and money supply instruments and the use of budgetary measures has been restrained by binding rules aimed at avoiding the creation and maintenance of excessive public deficits.

The need for fiscal discipline, in a context where fiscal policies stood at national level, was justified by the potential external negative effects that could result from excessive deficits run by any participant in the eurozone. These effects (e.g., De Grauwe, 2005) included a possible increase in the interest rate of the EMU, leading to possible pressures on the European Central Bank (ECB) to implement a more expansionist monetary policy, thus leading to an increase in inflation. Fundamentally, European fiscal rules have been put in place because of the need to keep price stability (e.g., Baldwin and Wyplosz, 2004).

The above-mentioned rules included a maximum ceiling for the deficit to GDP ratio (3%) and for the public debt to GDP ratio (60%) and were consecrated by the Maastricht Treaty (1992). Further, the need for fiscal discipline has been improved by the rules of the Stability and Growth Pact (SGP), which assumed a budgetary equilibrium (or even a small budgetary superavit) as the fundamental goal in terms of the medium-run, considering it the adequate way to preserve some margin for manoeuvre for fiscal policy in the case of a negative shock. The SGP went further on fiscal discipline, establishing concrete sanctions to be applied to countries that maintain excessive deficits and do not comply with Council recommendations (European Council, 1997).

Since the beginning, these rules have been the main object of discussion among academics and politicians with regard to fiscal policy in the eurozone. Positions vary from those who support them, possibly calling for small changes (e.g., Buti and Giudice, 2002;
Begg et al., 2004; Buti et al., 2005), to those who would prefer a federalisation of fiscal policy (e.g., Goodhart, 1990) or, if not possible, an important degree of autonomy for every national fiscal policy (e.g., Buiter et al., 1993; Maillet, 1992), and to those who, strongly supporting the case for fiscal discipline, require major reforms for the rules set in the case of the EMU (e.g., Casella, 1999; Buiter and Grafe, 2002; Pisani-Ferry, 2004; Wyplosz, 2005).

Among the arguments put forward in favour of major changes in the original SGP,\(^1\) we find the idea that fiscal rules would differ according to the level of development of the Member-Countries and their economic dimension.

On the one hand, the goal of economic and social cohesion would require stronger rates of the real product in the less developed countries, which in turn would require stronger public intervention and would be compatible with temporary public deficit and debt ratios higher than those proposed in the SGP. Moreover, several authors (including Mills and Quinet, 2001; Brunila, 2002; or Creel, 2003) have suggested the substitution of current rules for the “golden rule”, allowing the creation of public debt to face public investment expenses, or the introduction of rules concerning public expenses (and not strictly the public deficit). In particular, some expenses related to public investment would be excluded from the calculus of the relevant public deficit concerning the application of the “3 per cent” rule.

On the other hand, the above mentioned external spillovers resulting from excessive deficits would vary according to the economic dimension of the country. It would be expected that only big countries, such as Germany, would affect financial markets and price stability in a relevant way.

\(^1\) For a more complete description of the critics and doubts concerning the European fiscal rules, see, for instance, Buiter et al., 1993, and, for a recent assessment, Buiter, 2005.
The debate is not yet closed, but it may already have made a relevant contribution to the recent SGP reform (European Council, 2005), which may have made European fiscal rules more flexible. In particular, the “new” SGP allows for a growing number of circumstances that lead to a non-automatic application of the sanctions, namely considering a diversified kind of public expenses that may justify the non-compliance to the “3 per cent” rule. As far as the present paper is concerned, it is relevant to note that, within that set, expenses regarding R&D are included.

In order to analyse whether or not such kind of public expenses should be treated differently and whether or not European fiscal rules should differ between countries, we consider a standard economic structure in endogenous R&D-growth theory, for two countries that compose a monetary union. In each country, the production of perfectly competitive final goods uses institutions and labour together with a continuum set of country specific quality-adjusted intermediate goods. Intermediate goods, in turn, use designs (resulting from R&D activities) under monopolistic competition. The production function, in which the complementarity of inputs, in each country, is coupled with substitutability between countries, is adapted from the horizontal R&D growth models developed by Kiley (1999) and Acemoglu and Zilibotti (2001).

As a result of the close relationship between the production of intermediate goods and R&D, this one can be encouraged either by a direct subsidy or through a subsidy to the production of intermediate goods. Such policies have a negative impact on the fiscal budget of each country and that situation may lead to adverse consequences, such as those prevented by the SGP.

\[\text{\footnotesize\textsuperscript{2}}\] Namely using the concept of flexibility included in the set of criteria that Kopits and Symansky, 1998, defined for “optimal” fiscal rules.
However, these policies may reduce the technological-knowledge gap between the two countries and, through this, increase the competitiveness of the less developed one. In this case, they would be fundamental for an increase in the economic convergence within the union and, in particular, for the economic growth performance of the poorer country, which could justify different fiscal rules among countries. As apparent above, this will be the focus of the present work.

In any of these cases, such policies also have a bearing on the demand for labour. Additionally, while affecting not only the level but also the technological-knowledge gap between countries, they also have an impact on the relative demand for labour in each country and, thus, on inter-country wage inequality. This important topic of research in recent literature (e.g., Wood, 1998; and Aghion et al., 2003) will also be discussed, as the model easily allows it, although it will not be the central focus of the paper.

By assumption, countries differ in three features. The first feature relates to economic dimension, measured by labour endowments: the one with higher active population is called Big, the other one is called Small. The second feature concerns domestic institutions, which are more advanced in the Big-country. The third feature relates to the domestic quality indexes measuring technological knowledge, which are higher in the Big-country. The latter feature is an endogenous consequence of the other two and measures the level of development of each country.

The paper is structured as follows. Section 2 describes the model. Section 3 determines the equilibrium conditions. Section 4 analyses the effects of a governmental intervention. Finally, section 5 offers some concluding remarks.

2. The model

2.1. Final-goods sector
Each final good $n \in [0, 1]$ is produced by one of two countries, the Small-country, $S$, and the Big-country, $B$. The former (latter) brings institutions, $A_S$ ($A_B$), and labour, $L_S$ ($L_B$), together with a continuum set of $S$ ($B$) specific quality-adjusted intermediate goods, indexed by $j \in [0, J]$ ($j \in [J, 1]$). The output of $n$, $Y_n$, at time $t$ is,

$$Y_n(t) = \left[ \int_0^J \left( q^{x_j(n)} x_n(k, j, t) \right)^{1-\alpha} \left( 1-n \right) A_S^{1-\alpha} L_{S,n}^\alpha \right]^{1-\alpha} \left[ \int_0^J \left( q^{x_j(n)} x_n(k, j, t) \right)^{1-\alpha} \left( n A_B^{1-\alpha} L_{B,n}^\alpha \right)^{1-\alpha} \right].$$  \hspace{1cm} (1)

The integrals denote the contribution of intermediate goods to production. In the Schumpeterian tradition, the quantity of each $j$, $x_n$, used in the production of the final good $n$ is quality-adjusted; i.e., the quality upgrade is $q > 1$, and $k$ is the top-quality rung at time $t$. The term $1-\alpha$ is the aggregate intermediate-goods input share.

The second and fourth terms on the right-hand side of (1) can be interpreted as representing the role of the labour to production, respectively, in the Small-country and in the Big-country. These terms include the labour levels of each country, where, by assumption, $L_B > L_S$. The term $A$ is an exogenous variable representing the level of productivity, dependent on country’s institutions. As $B$’s institutions are, by hypothesis, more advanced, we consider $A_B > A_S > 1$, which means that an absolute productivity advantage of $L_B$ over $L_S$ is accounted. A relative productivity advantage of either type is captured by $(1-n)$ and $n$, which implies that $L_S$ ($L_B$) is relatively more productive in final goods indexed by smaller (larger) $n$s. The parameter $\alpha \in ]0, 1[$ represents the labour input share.

Finally, as we will see below, at each time $t$ there is a competitive equilibrium threshold final good $\pi$, where the switch from one country to the other becomes advantageous. An increase in $\pi$ would mean a larger space for production in country $S$, thus appearing as a measure of its relative competitiveness.

Due to zero profit equilibrium by producers of $n \in [0, 1]$, the demand for the top-quality of $j$ by the producer of $n$ is
where: $p_n$ and $p(j)$ are, respectively, the prices of $n$ and $j$. A higher $p_n$ increases the marginal revenue product of the factors, encouraging firms to rent more intermediate goods. A higher $L_{S,n}$ or $L_{B,n}$ implies that more labour is used with intermediate goods, raising demand. Finally, a higher $p(j)$ means lower demand, since the demand curve for intermediate goods is downward sloping.

Plugging (2) into (1), we have the supply of $n$:

$$X_n(k,j,t) = \left[ \frac{p_n(t)(1-\alpha)}{p(j,t)} \right]^{1/\alpha} q_{(j,t)}^{(1-\alpha)/\alpha} \left\{ (1-n) A_n^{1/\alpha} L_{S,n}(t), \text{if } 0 < j \leq J; \text{ i.e., } n \text{ is produced in } S \right\} n A_n^{1/\alpha} L_{B,n}(t), \text{if } J < j \leq I; \text{ i.e., } n \text{ is produced in } B .$$

(2)

(4) are aggregate quality indexes, measuring the technological knowledge in the country-specific range of intermediate goods.

Let us define $G \equiv Q_B/Q_S$. $G$ accounts for the relative technological-knowledge level of the Big’s specific intermediate goods, giving an adequate measure of the technological-knowledge gap between countries or, in other words, of the economic development gap.

As will be shown later, an endogenous relevant result is that $G^{-1} < 1$, as $Q_S < Q_B$, because the Small-country has less labour and worse institutions. It will also be shown that this result allows us to analyse whether a country specific governmental intervention may

\[\text{(3)}\]

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3 Thus, $L_{S,n}$ and $L_{B,n}$ are effectively the markets for new technologies, since monopolists can only sell intermediate goods to the producers of final goods employing labour.

4 As we will see below, the profit maximising by monopolist producers implies that $p(j)$ is independent of $j$. 

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improve the Small-country situation, thus questioning the existence of a one size fits all fiscal rule.\textsuperscript{5}

We define the aggregate output of the union, i.e., the composite final good, as:

\begin{equation}
Y(t) = \int_{0}^{1} p_a(t) Y_a(t) \, dn = \exp \left( \int_{0}^{1} \ln p_a(t) \, dn \right) \exp \left( \int_{0}^{1} \ln Y_a(t) \, dn \right) = \exp \left( \int_{0}^{1} \ln Y_a(t) \, dn \right),
\end{equation}

where we normalise its price at each time \( t \) to one (numeraire). Resources of the union, \( Y \), that are not consumed, \( C \), are used in the production of intermediate goods, \( X \), and in the R&D sector, \( R \); i.e., \( Y = X + R + C \).

\subsection*{2.2. Intermediate-goods sector}

Since \( Y \) is the input in the production of \( j \in [0, 1] \) and final goods are produced in perfect competition, the before-subsidy marginal cost of production of \( j \in [0, 1] \) is 1, regardless of the country. Assuming that the government of each country can subsidise the production of \( j \) by paying an ad-valorem fraction, \( z_x \) (more specifically, \( z_{x,S} \) in \( S \) and \( z_{x,B} \) in \( B \)), of each firm’s cost, the after-subsidy marginal cost of producing \( j \) is \((1 - z_x)\); i.e., \((1 - z_{x,S})\) in \( S \) and \((1 - z_{x,B})\) in \( B \).

Following Romer (1990), \( j \) embodies a costly design (created in the R&D sector), which is recovered if profits at each date are positive for a certain time in the future. This is assured by a patent law (i.e., by a system of intellectual property rights), which protects each leader firm’s monopoly, while at the same time, almost without costs, disseminating acquired technological knowledge to other firms.

The profit-maximisation price of the monopolistic firms yields the constant over \( t \), across \( j \) and for all \( k \) mark-up \( p(k, j, t) = p = (1 - z_x) / (1 - \alpha) \), which, with \( z_x < \alpha \), is in fact a

\begin{footnote}
Alternatively, it would be possible to analyse the effects of a supranational intervention, using a common budget, which grants a higher level of subsidies for the less developed country in order to produce a higher level of economic convergence within the union.
\end{footnote}
mark-up over 1. Without any change in government intervention, this mark-up is stable over \( t \), across \( j \) and for all \( k \). This symmetry is thus dictated by the way in which each \( j \) enters (1) and by the fact that all intermediate good producers use the same input.

Since the leader firm is the only one legally allowed to produce the top-quality, it will use pricing to wipe out sales of lower quality. Depending on whether \( q (1-a) \) is greater or lesser than the marginal cost of production, it will respectively use the monopoly pricing \( p = (1-z_x)/(1-\alpha) \) or the limit pricing \( p = q (1-z_x) \) to capture all the market. As in Grossman and Helpman (1991, Ch. 4), it is assumed that limit pricing strategy is used by all firms. Since the lowest price that the closest follower can charge without negative profits is \( (1-z_x) \), the leader can successfully capture all the market by selling at a price slightly below \( q(1-z_x) \), because \( q \) represents the quality advantage over the closest follower.

2.3. R&D sector

The outcomes of R&D are designs, which improve the quality of intermediate goods and, thus, the aggregate quality indexes in (4), while creatively destroying the profits from previous improvement (e.g., Aghion and Howitt, 1992), as the previous best quality loses that status.

In intermediate good \( j \) at time \( t \), a firm engaged in R&D that uses \( y(k, j, t) \) flow of \( Y \) is successful in upgrading the next quality, \( k(j, t)+1 \), with instantaneous probability

\[
I(k, j, t) = y(k, j, t) \cdot \beta q^{k(j,t)} \cdot \xi^{-1} q^{-a^*k(j,t)},
\]

where:

(i) the R&D activity is located in \( S(B) \) if \( 0 \leq j \leq J \) \((J < j \leq 1)\); (ii) \( \beta(q^{k(j,t)} \cdot \beta > 0 \), is the positive learning effect of accumulated public technological knowledge from past R&D in
is the adverse effect caused by the increasing complexity of quality improvements in \( j \) (e.g., Kortum, 1997, and Dinopoulos and Segerstrom, 2005).\(^7\)

The positive learning effect is thus modelled in such a way that, together with the complexity effect, it totally offsets the positive influence of the quality rung on the profits of each intermediate good leader firm, as we can see below. This is the technical reason for the presence of the production function parameter \( \alpha \) in (6).

As mentioned earlier, we will allow any of the governments to subsidise R&D activities directly, by means of an ad-valorem subsidy \( z_r \), which can be country-specific (i.e., \( z_{r,S} \) in \( S \) and \( z_{r,B} \) in \( B \)).

### 2.4. Consumers

A time-invariant number of heterogeneous individuals in the union (also as in each country) – continuously indexed by \( a \in [0, 1] \) – decide the allocation of income, which is partly spent on consumption of the composite final good, and partly lent in return for future interest. For simplicity, we consider an exogenous threshold individual \( \alpha \), smaller than 0.5, since \( L_B = \int_0^1 da > L_S = \int_0^\alpha da \): individuals \( a > \alpha \) are located in \( B \), whereas individuals \( a \leq \alpha \) are located in \( S \).

The infinite horizon lifetime utility of the individual \( a \) is the integral of a discounted constant elasticity of substitution (CIES) utility function,
\[ U(a,t) = \int_0^\infty \left[ \frac{c(a,t)^{1-\theta} - 1}{1-\theta} \right] \exp(-\rho t) \, dt , \]

where: (i) \( c(a,t) \) is the amount of consumption of the composite final good by the individual \( a \), at time \( t \); (ii) \( \rho > 0 \) is the homogeneous subjective discount rate; and (iii) \( \theta > 0 \) is the inverse of the inter-temporal elasticity of substitution.

The budget constraint of individual \( a \) equalises income earned to consumptions plus savings, at each \( t \). Savings consist of accumulation of financial assets \(- K\), with return \( r \) – in the form of public debt owned by individuals and in the form of ownership of the firms that produce intermediate goods in monopolistic competition.\(^8\) The budget constraint, expressed as savings + consumptions = income, is

\[ \dot{K}(a,t) + c(a,t) = [1 - \tau_K] r(t) K(a,t) + \begin{cases} [1 - \tau_{w,g}] w_g(a,t), & \text{if } a \leq \bar{a} \\ [1 - \tau_{w,b}] w_b(a,t), & \text{if } a > \bar{a} \end{cases} , \]

where:

(i) \( K(a,t) \) is thus the total asset holdings of the individual \( a \), with return \( r \); (ii) \( w(a,t) \) is the wage of the individual \( a \), at time \( t \); (iii) \( \tau_K \) and \( \tau_w \) are the ad-valorem taxes on assets and wages, respectively, which may be used by the government for fiscal policies purposes (in particular, as a means of financing, at least partially, the costs of the above mentioned subsidies); (iv) \( w \) and \( \tau_w \) may differ between countries, but not \( \tau_K \);\(^9\) (v) \( r \) is the same within the union, as a natural consequence of the monetary union.\(^10\)

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\(^8\) The value of these firms, in turn, corresponds to the value of patents in use.

\(^9\) Note that wages are endogenously determined, while taxes are exogenous parameters. We assume that \( \tau_K \) is the same in the two countries while \( \tau_w \) may differ, as in the case of the European Union, the mobility of capital is largely greater than the mobility of labour, thus determining a higher degree of harmonisation in the case of taxation over financial assets revenues.

\(^10\) Also note that due to arbitrage in the domestic assets market, \( r \) depends on \( t \), but is independent from the type of labour.
Maximising (7) subject to (8) yields the growth rate of consumption, which is independent of the individual and is the standard Euler equation:

\[ \hat{c}(a,t) = \hat{c}(t) = \frac{1}{\theta} \ln \left( 1 - \tau_k \right) r(t) - \rho \],

where \( C(t) = \int_0^1 c(a,t) \, da \). (9)

Since, in addition, to firms and individuals, both economies can also be influenced by domestic government policies, in order to finalise the characterisation of both economies, a description of the government’s budget is in order.

2.5. Government

In this model, the government of each country may intervene by imposing taxes on wages and/or on financial assets and by subsidising the production of intermediate goods and/or R&D activities. If necessary, the government may run a public deficit by issuing public debt sold to individuals.

The budget superavit, \( BuS \), of \( S \) and \( B \) is given respectively by:

\[
(1 + r(t)) BuS_s(t) = \tau_k \, r(t) \int_0^\pi K(a,t) \, da + \tau_{\pi, S} \int_0^\pi w(a,t) \, da - z_{s,S} X_s(t) - z_{R,S} R_s(t) - r(t) D_s(t) ;
\]

\[
(1 + r(t)) BuS_B(t) = \tau_k \, r(t) \int_\pi^1 K(a,t) \, da + \tau_{\pi, B} \int_\pi^1 w(a,t) \, da - z_{s,B} X_B(t) - z_{R,B} R_B(t) - r(t) D_B(t) .
\]

Where: (i) \( X_s \) (\( X_B \)) represents the resources devoted to intermediate goods production in \( S \) (\( B \)); (ii) \( R_s \) (\( R_B \)) represents the resources devoted to R&D in \( S \) (\( B \)); and (iii) \( D_s \) (\( D_B \)) represents the public debt in \( S \) (\( B \)). Thus, the first and second terms on the right-hand side represent government tax revenue from assets income and from labour income, respectively, while the third and fourth terms represent government expenditure on subsidies for intermediate goods and for R&D, respectively, and the last term relates to interest paid on public debt.\(^{11}\)

\[^{11}\text{We know that } BuS(t) = Taxes(t) - Subsidies(t) - r(t) D(t - dt) \text{ and } D(t) = D(t - dt) - BuS(t) , \text{ which gives } D(t - dt) = D(t) + BuS(t) , \text{ leading to equations (10a, b).} \]
We will be particularly interested in the effects of higher levels of subsidies in the less developed country, regarding an eventual convergence towards the level of development of the other country. Such an effect would become an argument in favour of different fiscal rules among countries in the union, namely a temporary authorisation for higher ratios between public deficit and GDP in the less developed countries.

3. Equilibrium

The dynamic general equilibrium resulting from optimal decentralised (*laissez-faire* or private) behaviour can be described by the path of both country-specific aggregate quality indexes, $Q_S$ and $Q_B$, towards the steady state.

3.1. Equilibrium for given technological knowledge

With perfect competition in final goods, the competitive advantage of either country on the production of the $n^{th}$ final good depends on the relative productivity related with the quality of national institutions, $(A_b/A_s)^{\mu_\alpha}$, and on the price of the country-specific labour, as well as on the relative productivity and prices of the intermediate goods, because of complementarity in production.

The prices of labour rely on the quantities, $L_B$ and $L_S$. In relative terms, the productivity-adjusted quantity of $L_B$ in production is $A_b^{\alpha_3}A_s^{\alpha_3}L_BL_S^{-1}$. As for the productivity and prices of intermediate goods, they depend on complementarity with either labour, $L_B$ or $L_S$, on the technological knowledge in the country-specific range of intermediate goods and on the mark-up. These determinants are summed up in the aggregate quality indexes, $Q_B$ and $Q_S$, in (4).
The endogenous threshold final good $\bar{n}$ follows from equilibrium in the inputs markets and relies on the determinants of the competitive advantage in final goods.\(^{12}\)

$$
\bar{n}(t) = \left\{ 1 + \left[ G(t) \frac{A_B^{n-1} L_B}{A_S^{n-1} L_S} \right]^{\frac{1}{2}} \right\}^{-1}.
$$

(11)

It can be related to prices bearing in mind that it is indifferent to produce the threshold final good in $B$ or $S$. This yields the ratio of index prices of final goods produced in each country,

$$
\frac{p_B(t)}{p_S(t)} = \left[ \frac{\bar{n}(t)}{1-\bar{n}(t)} \right]^\alpha,
$$

where:

$$
\begin{align*}
p_B &= p_s (1-n)^\alpha = \exp(-\alpha) \bar{n}^{-\alpha} & \text{since } \exp\int_0^1 \ln p_s \, dn = 1. \quad (12)
\end{align*}

Equation (11) shows that a higher economic development gap, $G$, a larger relative supply of labour, $L_B/L_S$, and/or a higher relative productivity concerning the quality of national institutions, $A_B/A_S$, results in a higher fraction of final goods produced in $B$, thus in a small $\bar{n}$. By (12), small $\bar{n}$ implies a low relative price of final goods produced by $B$. In this case, the demand for $B$ specific intermediate goods is relatively low, which discourages R&D activities aimed at improving their quality, as we can see below.

The equilibrium aggregate resources devoted to intermediate-goods production, $X=X_B+X_S$, and the equilibrium aggregate output, $Y=Y_B+Y_S$, i.e., the composite final good in the union (5), are expressible as a function of the currently given factor levels,

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\(^{12}\) This means that the competitive equilibrium threshold $\bar{n}$, arises from: (i) profit maximisation by perfectly competitive producers of final goods; (ii) profit maximisation by monopolist firms producing intermediate goods; and (iii) full-employment equilibrium in factor markets, given the supply of labour and the current state of country-specific technological knowledge. Thus, $B$ produces final goods $n > \bar{n}$ and $S$ produces final goods $n \leq \bar{n}$.
\[ X(t) \equiv \int_0^1 x_n(k,j,t) \, dj \, dn = \begin{cases} \displaystyle X_b(t) = \left[ \frac{p_{b}(t) A_b (1-\alpha)}{q(1-z_{b,s})} \right]^{1/\alpha} L_b Q_b(t) + \left[ \frac{p_{b}(t) A_b (1-\alpha)}{q(1-z_{b,b})} \right]^{1/\alpha} L_b Q_b(t) \end{cases} ; \tag{13a} \]

\[ Y(t) \equiv \int_0^1 p_s(t) Y_s(t) \, dn = \begin{cases} \displaystyle Y_s(t) = \left[ \frac{1-\alpha}{q(1-z_{s,s})} \right]^{\frac{1-\alpha}{\alpha}} p_{s}^{1/\alpha}(t) A_s^{1/\alpha} L_s Q_s(t) + \left[ \frac{1-\alpha}{q(1-z_{s,b})} \right]^{\frac{1-\alpha}{\alpha}} p_{s}^{1/\alpha}(t) A_s^{1/\alpha} L_b Q_b(t) \end{cases} . \tag{13b} \]

Equation (13b) shows clearly that: (i) economic growth is driven by the technological-knowledge progress, reflected in the aggregate quality indexes;\(^\text{13}\) (ii) the contribution of \( B \) for the composite final good is higher than the contribution of \( S \), since, by assumption, \( L_B > L_S \) and \( A_B > A_S \), and, as an endogenous result of the model, \( Q_B > Q_S \) – as will be shown later.\(^\text{14}\)

The price paid for a unit of labour, \( w \), is equal to its marginal product. From (13b), the equilibrium measure of inter-country wage inequality, \( W \), is, at each time \( t \):

\[ W = \frac{w_B}{w_S} = \left( G(t) \frac{A_B^{\alpha-1} L_S}{A_S^{\alpha-1} L_B} \right)^{\frac{1}{2}} . \tag{14} \]

\(^{13}\) As will be shown later, in transitional dynamics towards the steady state, the growth rate is higher in the country with stronger technological knowledge progress.

\(^{14}\) Since \( S \) is not too backward (i.e., an appropriate taxonomy for our \( B \) and \( S \) countries would be developed \textit{versus} developing, rather than developed \textit{versus} underdeveloped), it is predictable that inter-country differences in prices of final goods are of second order. Moreover, in the context of a monetary union, with single currency and common market, prices of tradable goods tend to be very similar, as well as national inflation rates.
Equation (14) shows that inter-country wage inequality grows with: (i) endogenous accumulation of technological knowledge by $B$; (ii) exogenous improvement in $B$’s institutions; and (iii) exogenous decrease in labour endowments of $B$.\textsuperscript{15}

3.2. Equilibrium R&D

The expected current value of the flow of profits to the monopolist producer of intermediate good $j$, $V(k, j, t)$,\textsuperscript{16} relies on: (i) profits at each time, $\Pi(k, j, t)$, given by

\begin{equation}
\Pi(k, j, t) = \left\{ \begin{array}{l}
L_S (1 - z_{s,s}) \alpha^{-1}(\alpha-i) (q-1) \left[ \frac{p_s(t) A_s (1-\alpha)}{q} \right] \alpha^{-1} q^{k(j,t) \alpha^{-1}(i-\alpha)} \\
L_B (1 - z_{s,b}) \alpha^{-1}(\alpha-i) (q-1) \left[ \frac{p_b(t) A_b (1-\alpha)}{q} \right] \alpha^{-1} q^{k(j,t) \alpha^{-1}(i-\alpha)} 
\end{array} \right.,
\end{equation}

in $S$ and $B$, respectively; (ii) the given equilibrium interest rate; and (iii) the expected duration of the flow, which is the expected duration of the successful research’s technological-knowledge leadership. Such duration, in turn, depends on the probability of a successful R&D. The resulting expression for $V(k, j, t)$ is:

\begin{equation}
V(k, j, t) = \frac{\Pi(k, j, t)}{r(t) + I(k, j, t)}.
\end{equation}

Hence, the expected income generated by the successful research on rung $k^\text{th}$ at time $t$, $V(k, j, t) r(t)$, equals the difference between profit flow, $\Pi(k, j, t)$, which is paid out as dividends, and the expected capital loss, $V(k, j, t) I(k, j, t)$, which will occur when rung $k^\text{th}$ is replaced by a new one. Thus, $r + I$ is the effective discount rate of the successful R&D.

Under free-entry R&D equilibrium in each country the expected returns are equal to resources spent,

\textsuperscript{15} Since we consider inter-Union labour immobility, it is not possible to solve the problem of inter-country wage inequality through immigration from $S$ to $B$.

\textsuperscript{16} I.e., $V(k, j, t)$ is the market value of the patent or the value of the monopolist firm, owned by consumers.
\[ I(k, j, t) V(k+1, j, t) = \begin{cases} (1-z_{r,S}) y(k, j, t), & \text{if } 0 < j \leq J; \text{i.e., in } S \\ (1-z_{r,B}) y(k, j, t), & \text{if } J < j \leq 1; \text{i.e., in } B \end{cases} \]  

(17)

where \( z_r \) is a governmental ad-valorem subsidy to R&D and it can be country-specific.

The equilibrium can be translated into the path of the technological knowledge. The following expression results for the equilibrium country-specific technological-knowledge growth rate in \( S \) and \( B \), respectively:

\[
\hat{Q}_s(t) = \frac{\beta}{\xi} \frac{1 - z_{r,S}}{1 - z_{r,S}} \left( q - 1 \right) \left( L_s \left[ p_s(t) A_s (1-\alpha) \right]^{\alpha-1} - r(t) \right) \left[ q^{- \alpha (1-\alpha)} - 1 \right].
\]

(18a)

\[
\hat{Q}_b(t) = \frac{\beta}{\xi} \frac{1 - z_{r,B}}{1 - z_{r,B}} \left( q - 1 \right) \left( L_b \left[ p_b(t) A_b (1-\alpha) \right]^{\alpha-1} - r(t) \right) \left[ q^{- \alpha (1-\alpha)} - 1 \right].
\]

(18b)

In (18a) and (18b), the terms in large brackets are the equilibrium country-specific probability of successful R&D, \( I_S \) and \( I_B \), given \( r \), \( p_S \) and \( p_B \), which turns out to be independent of \( j \) and \( k \), due to the removal of scale of technological-knowledge effects.\(^{17}\)

Equations (18a) and (18b) indicate that subsidies may improve technological knowledge and, through it, the country’s level of development.

Substituting \( p_S \) and \( p_B \) in (18a, b) for the respective expressions in (12) and equalling (18a) and (18b), we find the equilibrium value of \( \bar{\pi} \). After that, the equilibrium levels of \( p_S \) and \( p_B \) are also revealed.

The equilibrium aggregate resources devoted to R&D, \( R \), at each time \( t \), are

\[
R(t) = \begin{cases} \int_0^t y(k, j, t) dj = \zeta \beta^{-1} Q_s(t) L_s I_s(t), & \text{in } S \\ \int_j^1 y(k, j, t) dj = \zeta \beta^{-1} Q_b(t) L_b I_b(t), & \text{in } B \end{cases}
\]

(19)

\(^{17}\) As stated before, the positive effect of the quality rung on profits and on the learning effect is exactly offset by its effect on the complexity cost.
Hence, (19) shows that more resources devoted to R&D in each country (and thus in the union) are needed to offset the greater difficulty of R&D when the technological knowledge in the country-specific range of intermediate goods rises.

3.3. Steady state

Since the aggregate output has constant returns to scale in inputs $Q_S$ and $Q_B$, and $Y, X, R$ and $C$ are all multiples of $Q_S$ and $Q_B$, the constant and unique steady-state endogenous growth rate, which through the Euler equation (9) also implies a constant steady-state interest rate, $r^*(=r_s^* = r_b^*)$, designed by $g^*(= g_s^* = g_b^*)$ is:

$$
g^* = \hat{Q}_s = \hat{Q}_b = \hat{Y} = \hat{X} = \hat{R} = \hat{C} = \hat{c} = \frac{1}{\theta} \left( 1 - \tau_k \right) r - \rho \right] \Rightarrow \hat{G} = \hat{p_s} = \hat{p_b} = \hat{n} = 0.
$$

Therefore, $r^*$ is obtained by setting the growth rate of consumption in (9) equal to the growth rate of technological knowledge in (18a, b) and using the previously determined equilibrium levels of $p_s$ and $p_b$. Then, $g^*$ results from plugging $r^*$ into the Euler equation (9). Also from (20), we find that the inter-country wage inequality remains constant in steady state, since from (14) $\hat{w}_u - \hat{w}_l = \hat{Q}_u - \hat{Q}_l = \hat{G} = 0$.

4. Government intervention

Now, we solve numerically the transitional dynamics towards the steady state to illustrate the effect of government intervention on the country-specific technological knowledge. The stability properties of the transitional dynamics towards the steady state are block recursive, in the sense that we can first determine the stability of $G$ and then recursively characterise the behaviour of all the other variables.\(^{19}\)

\(^{18}\) Note that $C = Y - X - R$ and $Y, X$ and $R$ are multiples of $Q_S$ and $Q_B$, then $C$ also becomes a multiple of these aggregate quality indexes.

\(^{19}\) We solve the model numerically because the differential equation describing the path of $G$ is non-linear, also because we want to look at the path of adjustment of some fundamental variables.
Using these results, we analyse whether different fiscal rules may be needed (or not) in order to offset divergences in development among member-countries within a monetary union. We also check the path of inter-country final goods productive structure and, complementarily, the path of inter-country wage inequality.

Bearing in mind that $r$ is always unique, (18a) and (18b) can be used to get the differential equation needed to obtain the path of the technological-knowledge gap between countries, $G$, and then the behaviour of other variables can be characterised:

$$
\hat{G}(t) = \frac{\beta}{\xi} \left( \frac{q-1}{q} \right) \left( q^{\alpha-1} (1-\alpha) - 1 \right) (1-\alpha)^{\alpha-1} \exp(-1) \left\{ \frac{1-z_{s,s}}{1-z_{r,s}} \right\} \frac{A_g}{1-z_{s,s}} L_g \times \\
\times \left[ 1 + \left( G(t) \frac{A_g^{\alpha-1} L_g}{A_s^{\alpha-1} L_s} \right)^{\alpha-1} \right]^{-\alpha} \left( 1-z_{s,s} \right)^{\alpha-1} L_s \left[ 1 + \left( G(t) \frac{A_g^{\alpha-1} L_g}{A_s^{\alpha-1} L_s} \right)^{\alpha-1} \right]^{-\alpha}.
$$

(21)

Using the fourth-order Runge-Kutta classical numerical method, which solves (21) with suitable precision, the time path of technological-knowledge gap is displayed, bearing in mind the baseline parameter values and labour endowments in table 1.

**Table 1. Baseline values of exogenous variables and parameters**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Value</th>
<th>Variables</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_B$</td>
<td>1.40</td>
<td>$L_S$</td>
<td>1.00</td>
<td>$\beta$</td>
<td>1.00</td>
<td>$\rho$</td>
<td>0.02</td>
</tr>
<tr>
<td>$A_S$</td>
<td>1.00</td>
<td>$s_{m,s}$</td>
<td>0.00</td>
<td>$z$</td>
<td>6.00</td>
<td>$\alpha$</td>
<td>0.70</td>
</tr>
<tr>
<td>$L_B$</td>
<td>1.40</td>
<td>$t_{w,m}$</td>
<td>0.00</td>
<td>$\theta$</td>
<td>1.05</td>
<td>$q$</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Notes: (i) the baseline values are in line with our theoretical assumptions (for instance, $A_B > A_S$), Acemoglu and Zilibotti (2001), Connolly and Valderrama (2005), and to calibrate the union growth rate around 2.5%; (ii) we consider $\overline{a} = 0.42 = L_s$ and this value is normalised to 1; (iii) we start with no governmental intervention.

We assume that initially $G = 1.40$, $BuS = D = 0$ in both countries. For simplification, we will assume in all built scenarios that there are no taxes: one consequence of this...
assumption is that there will be no effects on the budget superavit of one country resulting from changes in the fiscal policy of the other.

Figure 1 below sums up the main results, by comparing paths of, respectively, \(1/G\), \(\bar{n}\) and \(W\), under no government intervention in any country (Scenario 0 or Sc 0) with the ones resulting from an exogenous increase at time \(t = 0\) of: (Scenario 1 or Sc 1) \(z_{x,S}\) (to \(z_{x,S} = 0.1\)); (Scenario 2 or Sc 2) \(z_{r,S}\) (to \(z_{r,S} = 0.1\)); (Scenario 3 or Sc 3) \(z_{x,S}\) and \(z_{r,S}\) (to \(z_{x,S} = z_{r,S} = 0.1\)); and (Scenario 4 or Sc 4) \(z_{r,B}\) (to \(z_{r,B} = 0.1\)). Figure 1 also displays the path of the public deficit to product ratio for \(S\) in scenarios 1, 2 and 3, and for \(B\) in scenario 4. Moreover, table 2 compares initial and final steady-state values of the main variables and initial and final values for the deficit to product ratio under the different scenarios.

Figure 1. Transitional dynamics of:

\begin{itemize}
\item[a.] The technological-knowledge gap, \(1/G\)
\item[b.] Threshold final good, \(\bar{n}\)
\item[c.] Inter-country wage inequality, \(W\)
\item[d.] Public deficit to Product ratio, \(BuS/Y\)
\end{itemize}
Table 2. Initial and final steady-state values of the main variables

<table>
<thead>
<tr>
<th></th>
<th>$1/G$</th>
<th>$\bar{\pi}$</th>
<th>$W$</th>
<th>$p_B$</th>
<th>$p_S$</th>
<th>$r$</th>
<th>$g$</th>
<th>$(BuS/Y)_B$</th>
<th>$(BuS/Y)_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0.714</td>
<td>0.360</td>
<td>1.272</td>
<td>0.678</td>
<td>1.016</td>
<td>0.046</td>
<td>0.024</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sc 0</td>
<td>0.578</td>
<td>0.336</td>
<td>1.414</td>
<td>0.661</td>
<td>1.066</td>
<td>0.046</td>
<td>0.025</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sc 1</td>
<td>0.654</td>
<td>0.350</td>
<td>1.329</td>
<td>0.671</td>
<td>1.036</td>
<td>0.047</td>
<td>0.026</td>
<td>0.00</td>
<td>$-0.028$</td>
</tr>
<tr>
<td>Sc 2</td>
<td>0.773</td>
<td>0.369</td>
<td>1.223</td>
<td>0.685</td>
<td>0.998</td>
<td>0.049</td>
<td>0.028</td>
<td>0.00</td>
<td>$-0.101$</td>
</tr>
<tr>
<td>Sc 3</td>
<td>0.877</td>
<td>0.384</td>
<td>1.148</td>
<td>0.699</td>
<td>0.971</td>
<td>0.051</td>
<td>0.029</td>
<td>0.00</td>
<td>$-0.132$</td>
</tr>
<tr>
<td>Sc 4</td>
<td>0.431</td>
<td>0.304</td>
<td>1.637</td>
<td>0.640</td>
<td>1.143</td>
<td>0.063</td>
<td>0.041</td>
<td>$-0.184$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Our results clearly indicate that with no governmental intervention (scenario 0) the inter-country technological-knowledge gap, $1/G$, (and thus the divergence in levels of economic development) would grow significantly. That would cause an increase in the inter-country wage inequality, $W$, and a decrease in the competitiveness of the small and less developed country, as measured by the decrease in $\bar{\pi}$.

A governmental intervention in $S$, by way of an increase in $z_{x,S}$ and/or $z_{r,S}$ (in relation to $z_{x,B}$ and/or $z_{r,B}$, respectively), attenuates the inter-country technological-knowledge gap (scenario 1) or even makes it possible for $S$ to reduce this gap (scenarios 2 and 3), as Figure 1a shows.

A greater $z_{x,S}$ increases the size of profits that accrue to the producers of intermediate goods in $S$ – see (15) –, while a greater $z_{r,S}$ decreases the cost of R&D in $S$ – see (17). In this way; an increase in $z_{x,S}$ and/or $z_{r,S}$ boosts the incentives to do R&D in $S$, thereby increasing the growth rate of its technological knowledge, $Q_S$ in (4), which in turn leads to a higher $1/G$. Until the new steady state, such bias increases the supply of intermediate goods in $S$, thereby increasing the number of final goods produced in this country – see (11) and Figure 1b – and lowering their relative price at least when compared to scenario 0 – see (12). This path continues towards the constant new steady-state level of the mentioned variables, implying that $1/G$ is attenuated (scenario 1) or reverted (scenarios 2
and 3), but at a decreasing rate until it reaches its new higher steady state level, as depicted in Figure 1a.

Due to complementarity between inputs in (1), changes in $W$ are closely related to the inter-country technological-knowledge gap, as (14) clearly shows. Since the exogenous increase in $z_{r,S}$ and/or $z_{r,S}$ attenuates or reduces this bias, the stimulus to the demand for labour in $S$ attenuates (scenario 1) or reduces (scenarios 2 and 3) the inter-country wage inequality, as Figure 1c illustrates.

As expected, the main adverse effect of the governmental intervention in $S$ is a continuous increase in its public deficit and in the ratio between the public deficit and the GDP.\(^{20}\) This increase is relatively under control in scenario 1, with an indirect subsidy to R&D through subsidising the production of intermediate goods, but assumes too high values in scenarios 2 and 3, from a certain moment of time, as Figure 1d shows.

It is worth noting that the external negative effects of this governmental intervention, in the small and less developed country (and thus the external negative effects of running public deficits in $S$), while in line with those expected (De Grauwe, 2005), are of very limited importance: table 2 shows a slight increase in the price level index in $B$ when compared to the scenario of non-intervention, and also in $S$ when compared with the initial situation (and only in scenario 1); table 2 also shows a slight increase in the interest rate within the union. In this way, it is possible to argue that financing the deficit running in the small country would have no major negative effects across the union.

Finally, we use scenario 4 to compare the effects arising from the same kind of governmental intervention in $S$ and $B$, allowing us to highlight the importance of economic dimension on external effects of running a (probably) excessive public deficit.

\(^{20}\) Note that the values of the public deficit would be reduced if taxes were also considered.
As would be expected, the growth rate of the union, $g$, increases more significantly in the case of a direct subsidy to R&D given by $B$.\textsuperscript{21} However, this intervention would lead to an important increase in the inter-country technological-knowledge gap, as well as in the inter-country wage inequality, and to a significant decrease in competitiveness in $S$.\textsuperscript{22}

External negative effects of the governmental intervention, again in line with what would be expected (De Grauwe, 2005), would be clearly stronger in this case, associated with the creation of an excessive public deficit in $B$, which would be rapidly unsustainable, as the path of the public deficit to product ratio illustrates (Figure 1d). In this case, we observe a significant increase in the interest rate of the union, mainly driven by the need to finance an increasing public deficit, also as an important growth in the price level index of $S$. In this way, it is possible to argue that financing the deficit running in the big country would have major negative effects across the union.

From the results of our model it seems to be possible to consider that:

(i) a governmental intervention subsidising (directly or indirectly) R&D led by a small country in a monetary union induces external negative effects of very little significance, at the same time generating some relevant internal effects, namely promoting economic convergence towards the level of development of the more developed countries;

(ii) a governmental intervention subsidising R&D led by a big country in a monetary union induces strong external negative effects, raising the interest rate and the price level

\textsuperscript{21} We choose to compare directly scenarios 2 and 4, as it resulted, in scenario 2, that a direct subsidy to R&D would produce stronger effects on $1/G$, $\pi$ and $W$. In any case, the same kind of comparison would be possible considering a subsidy to the production of intermediate goods.

\textsuperscript{22} The mechanism underlying these results is the same as the one explained for a governmental intervention in $S$. 


of the union, at the same time increasing the economic development disequilibrium between countries, i.e., significantly reducing the desired economic and social cohesion;

(iii) temporary differentiation of fiscal rules within the union, allowing less developed and small countries to have more fiscal margin for manoeuvre, arises as a valid argument, as it may promote social and economic cohesion with very small costs;

(iv) in this sense, a direct subsidy to R&D works more rapidly, even allowing a decrease in the economic development gap, but has stronger negative effects on fiscal discipline, so its use should be more restricted in temporal terms;

(v) one of the main important aspects of the recent SGP reform (European Council, 2005), namely the consideration of an exceptional character of some public expenses connected to the development of innovation and knowledge processes (thus possibly justifying an excessive deficit), seems to be reasonable;

(vi) in the same way, our results point to the relevance of some criticisms made to the European fiscal rules, while imposed homogeneously throughout the eurozone and not considering the composition of public expenses (e.g., Mills and Quinet, 2001) or the possibility of financing public investment through the increase of public debt (e.g., Creel, 2003).

4. Concluding remarks

The main purpose of this paper is to discuss one of the main questions raised by the emergence of the EMU, namely the existence (or not) of the same fiscal rules for different countries, i.e., independently of their economic dimension and level of development. In particular, the paper focuses on the possibility for small and less developed countries to run temporarily excessive deficits, in order to improve their economic development and the social and economic cohesion within the union.
To this purpose, we develop a dynamic general-equilibrium growth model with two countries forming a monetary union. Growth is driven by Schumpeterian-R&D applied to intermediate goods which complement labour in each country. In this context, we analyse the effects of a governmental intervention through subsidising (directly or indirectly) R&D activities and compare them to a situation with no governmental intervention.

An increase in this kind of subsidies in the less developed and small country, $S$, redirects R&D towards designs that improve the quality of its intermediate goods relatively more. This increases the productivity of these intermediate goods, which, in turn, diminishes the perfectly competitive domestic relative prices of final goods produced in $S$. Thus, through the price channel, the inter-country technological-knowledge gap is reduced, but at a decreasing rate until it reaches its new steady state. By connecting government intervention with the technological-knowledge progress, we relate government intervention with the path of relative economic development.

Such an intervention would lead to the creation and increase of public deficits. However, should they occur in a small country, their external negative effects would be of very little significance, while inducing some relevant internal effects, namely promoting economic convergence. Conversely, if an excessive deficit were run in the big country, strong external negative effects would occur, raising the interest rate and the price level of the union, and significantly reducing the level of economic and social cohesion.

These results suggest that temporary differentiation of fiscal rules within the union, increasing fiscal flexibility for the less developed and small countries, would promote social and economic cohesion with very small costs. In particular, the consideration of an exceptional character of some public expenses connected to the development of innovation and knowledge processes would be justified. This result goes in line with one of the main important aspects of the recent SGP revision.
In future research, we intend to further develop the analysis in order to consider the impact on our results arising from issues such as the possibility of cheaper R&D imitative activity by the less developed countries, the (more realistic) existence of taxation to attenuate the level of public deficits, or the possibility of positive external spillovers generated by fiscal policies.
References


