Wicksell, Keynes and the New Neoclassical Synthesis. Whither saving-investment imbalances?*

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Abstract

The New Neoclassical Synthesis (NNS) can be regarded as the newly established macroeconomic consensus. Prominent scholars also stress its ideal continuity with the great founders of modern macroeconomic thought, not only Keynes, as is quite natural, but also Knut Wicksell (e.g. Blanchard (2000), Woodford (2003)). These claims have prompted debates in two main directions. The first concerns their exegetic legitimacy, the second whether and to what extent a reconsideration of those original ideas may improve our understanding and policy of business cycles. After a brief overview of the former issue, this paper focuses on the latter, and in particular on the "Wicksell connection" of the NNS put forward by Woodford (2003). My main contention is that the hallmark, and the most enduring one, of Wicksell's and Keynes's approaches to business cycles was saving-investment imbalances (that is to say capital market failures and intertemporal disequilibrium in modern parlance) that are notably absent from the NNS, whereas the cornerstone of the NNS (sticky wages and prices) played a minor role in Keynes and no role at all in Wicksell. Section 2 clarifies some basic theoretical issues underlying the NNS. Section 3 presents a "Neo-Wicksellian" dynamic model whereby it is possible to assess, and hopefully clarify, some basic issues concerning the macroeconomics of saving-investment imbalances. Section 4 explores the dynamic properties of the economic system under different interest-rate determination mechanisms ranging from the Wicksellian bank mechanism to the LM model and the Taylor rule. Section 5 concludes.

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1. Introduction

The New Neoclassical Synthesis (NNS) can be regarded as the newly established macroeconomic consensus (Blanchard (2000)). Prominent scholars also stress the ideal continuity of the NNS with the great founders of modern macroeconomic thought, not only Keynes, as is quite natural, but also Knut Wicksell (e.g. Blanchard (2000), Woodford (2003)). The Keynesian origins are detected in the importance attached to real as well as nominal shocks on the demand side in the explanation of business cycles, and in the role of sticky nominal wages and prices in the transmission of monetary policy to real economic activity. The renewed "Wicksell connection"¹ is instead suggested by the (re)discovery of the nominal interest rate, vis-à-vis the "natural" interest rate prevailing at full-employment general equilibrium, as the pivot of rule-based monetary policy.

The claim of the Keynesian-Wicksellian roots of the NNS have prompted further developments in two main directions. The first concerns exegetic legitimacy, with reference to each author and with reference to their coexistence within the same frame. The second instead investigates whether and to what extent a reconsideration of those original ideas may improve our understanding and policy of business cycles (e.g. Boianovsky and Trautwein (2004), Laidler (2004)). After a brief overview of the former issue, this paper focuses on the latter. My main contention is that the hallmark, and the most enduring one, of Wicksell’s and Keynes’s approaches to business cycles was saving-investment imbalances (that is to say capital market failures and intertemporal disequilibrium in modern parlance, see Leijonhufvud (1981)) that are notably absent from the NNS, whereas the cornerstone of the NNS (sticky wages and prices) played a minor role in

¹ The reference is to the title of the celebrated paper by Leijonhufvud (1981) which examined the monetary theories of Keynes and the Keynesians in relation to the Wicksellian legacy. As we shall see, this paper is also quite relevant to the Wicksellian claims of the New Keynesians.
Section 2 clarifies some basic theoretical issues underlying the NNS. Section 3 presents a "Neo-Wicksellian" dynamic model whereby it is possible to assess, and hopefully clarify, some basic issues concerning the macroeconomics of saving-investment imbalances. Section 4 explores the dynamic properties of the economic system under different interest-rate determination mechanisms ranging from the Wicksellian bank mechanism to the LM model and the Taylor rule. Section 5 concludes.

2. The New Neoclassical Synthesis, Keynes and Wicksell

The NNS framework typically consists of three building blocks (e.g. Woodford (2003, ch. 4)). First, the household’s intertemporal optimal consumption plan, which depends on the entire vector of real interest rates on securities, and which can be altered as the central bank changes the opportunity cost of holding money vis-à-vis other assets – this relationship is labelled "IS". Non-zero (stock) demand for money (cash) is derived from the plan as a complement to the demand for consumption goods. Second, firms’ profit maximization with "sticky" price adjustments, which implies that shifts in the IS schedule, whether due to real or policy shocks, give rise to deviations of output (employment) and prices (inflation) from the respective optimal paths that would result with flexible prices – this second relationship is labelled PC (Phillips curve), if referred to the labour market, or AS (aggregate supply), if referred to the output market. Finally, the model is closed by an interest-rate rule (IR) that relates the monetary interest rate to deviations of output and prices from their flex-price path. In this view, the flex-price paths of output ("potential output") and inflation are the key normative target variables for the central bank2.

The key tenets of the NNS can be summarized as follows.

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2 Potential output, and the related level of employment, in turn may or may not correspond to the Walrasian allocations that would be realized by fully competitive markets. "Real" rigidities such as monopolistic competition, efficiency wages, etc., may be responsible for a lower level of potential output and a non-zero "natural rate of unemployment". Monopolistic competition is the most common assumption by the NNS.
1) Output and employment fluctuate in response to unexpected shocks in the determinants of aggregate demand and supply around the potential output path.

2) The economic system responds to shocks with variations in quantities in the short run because of "price stickiness", that is imperfections in the organization of goods and labour markets or disincentives of economic agents against price changes.

3) "Money matters": a) Monetary policy impulses have persistent real effects; b) the typical observed pattern is one where policy interventions (mainly activated by changes in administered rates and money-market rates) are followed by quick and large responses in short-term interest rates, monetary aggregates, total credit, and different measures of real economic activity, and by slow and delayed adjustment of different price indexes; c) real wages and profits are also procyclical with output after a monetary shock.

4) However, neither fiscal nor monetary interventions on aggregate demand are able to alter the level of potential output and the natural rate of unemployment permanently; their only effect would be to raise the average level of inflation.

5) Macroeconomic policy is best managed by means of "rules": policy makers should respond to a stable and transparent objective function such that fluctuations around the potential output and inflation paths are minimized, without tampering with the structural combination of potential output, unemployment and inflation.

It is apparent that the overall macroeconomic picture provided by the NNS is akin to the Old one that ruled the discipline in the Fifties and Sixties. Indeed, the NNS has grown as a branch of the so-called "New Keynesian Macroeconomics", the research programme launched in the early Eighties and aimed at micro-refurbishing some resilient building blocks of the Old Synthesis, such as sòuggish price changes, the sensitivity of consumption to current income, the investment accelerator, and the interest-elasticity of the demand for money (e.g. Greenwald and Stiglitz (1993)). Thus, to some interpreters (e.g. Blanchard (2000)), the evolution from the Old to the NNS may be regarded as an instance of technical progress applied to economic ideas.

It is widely held that technical progress has come from the "New Classical Revolution" of the Seventies and Eighties, culminating in the
methodology of dynamic stochastic general equilibrium (DSGE) based on intertemporally optimizing agents. Moreover, the NNS now has a clear and consistent counterpart consisting in a Walrasian (New Classical) DSGE model. The latter shows how resources would efficiently be allocated by an ideal system of continuously clearing markets with neither real nor nominal "frictions". On the other hand, the "New Synthetics" (Trautwein (2006)) argue that the New Classicals went too far in their faith in the Walrasian representation of market economies, which did not pass a number of empirical tests and of policy experiments. As was clarified by Hahn (1977) at the very beginning of the New Classical parable, Keynes's fundamental contribution was that there exist some critical non-Walrasian features in actual market economies. If not theoretically, these features have proved hard to ignore empirically.

Thus, the New Keynesians started investigating a new class of DSGE models in which agents optimize intertemporally in economies with some non-Walrasian features. Of course, the critical question was: What non-Walrasian features? From this standpoint, the NNS has focused on non-Walrasian goods and labour markets, characterized by a) transaction costs, which give rise to a positive demand for money, and b) imperfect competition, which determines less than Pareto optimal potential output, combined with c) other market imperfections that rationalize short-run nominal wage/price rigidity. Advocates of the NNS argue that the latter was in fact the key non-Walrasian assumption in the General Theory, and far more emphatically, they claim that filtering this assumption through the new methodology has produced successful business cycle models and policy prescriptions that outperform Old Keynesian as well as New Classical ones.

On the policy side, according to Woodford (2003) the NNS now offers central banks a "user friendly", though rigorous, theoretical framework which a) meets the current scientific canons, b) explains the evidence of real effects of monetary policy, c) grounds the rationale for macroeconomic stabilization on welfare analysis, d) is consistent with the principle of rule-

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3 Hahn also warned that not all possible non-Walrasian features are ipso facto classifiable as Keynesians.
4 Items sub c) are the crucial ingredient in the recipe since imperfect competition by itself does not lead to nominal wage/price rigidity. They range from small menu costs, to staggered contracts, to coordination failures. A good recent survey of these items from this point of view is the one by Van der Ploeg (2005).
based policy and allows central banks to assess alternative rules of systematic stabilization policy.

Particular interest and curiosity have been aroused by Woodford’s argument that, alongside with its revisited Keynesian legacy in the theory of business cycles, the NNS theory of monetary policy is also a modern restatement and refinement of the kernel of Wicksell’s theory as, for instance, set out in his most famous book, *Interest and Prices* (1898a)\(^5\). In Woodford’s view, this Wicksellian connection is substantial because (see e.g. pp. 49-55)

- interest-rate rules imply that the instrumental rate is anchored to the real interest rate that prevails when all macroeconomic state variables are at their target values (e.g. the intercept in the estimated Taylor rules): this real interest rate is best understood as Wicksell’s "natural rate of interest", namely the real rate of return to (marginal product of) capital that equals the consumers’ marginal rate of substitution along the potential output path
- interest-rate rules in general ensure determinate macroeconomic equilibria provided that they embody the so-called "Taylor principle", namely that the elasticity of the interest rate to excess inflation should be greater than one
- the Taylor principle also implies that excess inflation arises whenever the instrumental interest rate is below the level consistent with the natural rate, while curbing excess inflation requires the instrumental interest rate to be set above the level dictated by the natural rate.

These, Woodford argues, are also the key propositions of Wicksell’s interest-rate theory of inflation and monetary policy. No doubt, several points of coincidence exist. Yet the NNS theoretical framework differs from Wicksell’s substantially, so much so that one wonders whether the points of coincidence of the respective theories may survive to closer inspection.

First, Wicksell’s interest-rate theory of prices was elaborated not with reference to continuous intertemporal equilibrium of households’ asset

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\(^5\) The essential elements of his interest-rate theory of prices were set forth by Wicksell in a conference paper presented in Stockholm in 1898 and available in the English translation (1898b) in Wicksell’s collected works edited by E. Lindahl. An abridged version of the same paper was also presented by Wicksell before the Economic Section of the British Association in 1906, then published as "The Influence of the Rate of Interest on Prices" by *The Economic Journal*, 1907, 17, pp. 214-220.
stocks but as a development of a number of crucial aspects of *money creation and circulation* that were apparently unaddressed by the then dominant quantity-theoretic equation. In other words, Wicksell adopted not a *stock* approach but a *flow* approach to monetary analysis (Leijonhufvud (1981), Laidler (2004), Boianovsky and Trautwein (2004)).

Second, the NNS concentrates the non-Walrasian features of the economy in the goods (and/or labour) market whereas the capital market remains perfectly Walrasian. By contrast, in Wicksell’s book the interest-rate theory of prices appears in the context of a "pure credit economy" with no use of cash, that is one with Walrasian goods markets but with a capital market which is not Walrasian at all (Leijonhufvud (1981), p.160; see also Goodhart (2004) and Laidler (2004)). First and foremost, there are intermediaries between savers and investors, whose existence can only be due to some departures from the Walrasian paradigm. Moreover, all three actors on the capital market act with limited information, which is the reason why deviations of the market interest rate from the natural rate may arise. Woodford (2003, ch.2) even puts forward an alleged Wicksellian, entirely Walrasian, "cashless and frictionless" economy, the purpose being to show that, even in a world "where the concepts of money demand and supply become inapplicable" (p. 49), the central bank can still control the inflation rate by setting the interest rate on "base money". At first sight one wonders whether Woodford is taking us back to a chimerical Walrasian world with money (or, even more remarkably, without money but with a monetary authority). Indeed, as shown by Boianovsky and Trautwein (2004), it is hard to find in Woodford’s model a convincing explanation as to why anybody would hold non-zero stocks of base money (which are however forced into the household’s budget constraint) or any consistent proof of the central bank’s ability to set an independent interest rate on base money if its stock happens to be positive.

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6 In this model, base money is a riskless security that exchanges 1 to 1 units of account in all states in an Arrow-Debreu economy with complete asset markets where transactions are settled without actual currency exchanges by, say, a central computer that stores individuals’ book entries in a common unit. Asset pricing (interest rates determination) is derived from the representative household’s intertemporal optimization and the usual no-arbitrage condition. Note that the model does not preclude the existence of other riskless securities competing with base money.
Third, Wicksell’s cashless economy “is not a moneyless economy” (Laidler (2004), p.3) in that the key problem to be explained remains how a single agent can have his/her virtual account – i.e. his/her nominal purchasing power in number of “euros” – increased. Apart from selling goods and services, the only other way for an agent to increase his/her nominal purchasing power is to borrow. Consequently, the appropriate concept of money demand is the one expressed by borrowers, whereas the appropriate concept of money supply is the one expressed by lenders. Borrowers are investing firms and lenders are saving households, intermediated by banks. As long as non-bank agents borrow and lend one with the other, the total amount of nominal purchasing power in the economy is redistributed but cannot (need not) increase. The capital market finds its equilibrium at the natural rate of interest as determined by the “forces of productivity and thrift” that equate saving and investment at full-employment of resources. Yet, as soon as the banking system (central bank and private banks) comes into play, the latter proposition no longer necessarily holds. A private bank is in a position to grant additional nominal purchasing power to any of its depositors’ accounts with no one else in the economy undergoing an equivalent reduction. Likewise, a private bank can increase its own nominal purchasing (lending) power by borrowing from the central bank. Thus, the problem is that the banking system as a whole might both expand the total nominal purchasing power in the economy and allocate it at terms that differ from those dictated by full-employment saving-investment equilibrium.

In this framework, the connection between money creation and nominal income – the well-known “cumulative process” – is necessarily examined as out-of-equilibrium dynamics from one level of money and nominal income to another:

In Wicksell’s theory of the cumulative process, the maladjustment of the interest rate – the discrepancy between the market and the natural rate – is the central idea. It is also the idea that motivates the analysis of changes in the price level (or in nominal income) in terms of saving and investment. It is a simple but fundamental point. Use of the saving-investment approach to income fluctuations is predicated on the hypothesis that the interest rate mechanism fails to coordinate saving and investment decisions appropriately. This is where all the Wicksell Connection theories differ from Monetarism (Leijonhufvud (1981), p.132).
Leijonhufvud’s last sentence also applies perfectly to Woodford’s renewed attempt at a connection with Wicksell. Monetarism was developed on the grounds of continuous capital-market clearing in terms of asset stock equilibrium, farther and farther away from Wicksell’s macroeconomics of saving-investment imbalances. This line of theorizing has then been systematized and popularized in the DSGE methodology. As explained above, the NNS has fully embraced this methodology, though introducing the non-Walrasian features able to breathe new life into the monetary side of these models. Long-standing doubts and criticisms about whether these features are truly Keynesian, and whether they are necessary and sufficient to understand and master business cycles, are still unresolved (e.g. Van der Ploeg (2005)). Other scholars of Keynesian inspiration would instead stress the central role of saving-investment imbalances in the General Theory as well, and would therefore focus on different non-Walrasian features: in particular, those impinging upon financial markets and leading to intertemporal coordination failures (e.g. Minsky (1975), Leijonhufvud (1981), Greenwald and Stiglitz (1993), Solow and Hahn (1995)). Common to these views, though different in other methodological respects, is the idea that the older macroeconomics of saving-investment imbalances does offers guidance for consistent foundations of the interest-rate theory and practice of monetary policy precisely because it focuses on the interest rate as "the wrong price" in the system and lead us to investigate how the monetary authority can manage to "get it right".

3. The macroeconomics of saving-investment imbalances

To begin with, let us consider the flex-price DSGE benchmark adopted by the NNS. The key point is that the price vector along the GE path of the economy includes the relative price of factors (the real wage rate and the "natural interest rate" as dictated by "real" determinants), and the problem is how the economy reacts when the "wrong" price is the interest rate.

Suppose that at time $t$ the market real interest rate exceeds the natural one. Excess saving arises to which there corresponds excess supply

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7 Not without the complicity of the liquidity preference theory of the interest rate and of Keynesians themselves, according to Leijonhufvud.
in the goods market at time $t$, and, by intertemporal Walras Law, excess (planned) demand at time $t+1$. Note that the capital-market disequilibrium at time $t$, if uncorrected, must have an intertemporal disequilibrium effect on the goods (and labour) market at time $t+1$ even though prices (and wages) are perfectly "right" with respect to the natural interest rate. Excesses can only be eliminated by a combination of lower saving/higher investment at time $t$ and lower (than planned) consumption/higher (than potential) production at time $t+1$. Thus, it is required a combination of rise in unemployment/fall in inflation at time $t$, and rise in unemployment/rise of inflation at time $t+1$. The mix of adjustment in the inflation rate vis-à-vis the unemployment rate has nothing to do with the degree of price flexibility. The point is whether the nominal adjustment of the prices (and wages) can correct the "wrong" real rental price of capital. The only possibility is that deflation at time $t$ is followed by expected inflation at time $t+1$ to such an extent that the market real interest rate falls and realigns with the natural rate. Hence this a matter of the dynamic paths of adjustment and their forecast by agents. Consequently, as thoroughly explained by Leijonhufvud (1981), there are, or should be, two key logical implications of any Wicksellian (and, for that matter, Keynesian) model, namely

- "unemployment will not converge to its natural level unless the interest rate goes to its natural level – (...) the latter condition will not always be fulfilled" (p. 135)
- "with the interest rate at the right level, market forces should make unemployment converge to the natural rate – otherwise not" (p.136).

For historical precision, it should be stressed that in Wicksell original theory, conceived in terms of fully flexible prices, incidental mention was made of unemployment. It was Keynes with his principle of effective demand who understood that as long as the market real interest rate is "wrong" firms are constrained in the supply of goods and hence workers are constrained in their supply of labour no matter how deep deflation may be. Though this is pure general-equilibrium theory, the point apparently fell by

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8 "While Wicksell had refused to use his theory of cumulative processes for the explanation of industrial fluctuations, [it was] Lindahl [who] wanted to extend Wicksell's approach into a general theory of business cycle" (Boianovsky and Trautwein (2006, p. 8). Lindahl (1939) in fact included unemployment in his analysis foreshadowing the modern distinction between cyclical and structural unemployment (ibid, p.11).
the wayside in the Old Synthesis, and then in modern macroeconomics altogether (Leijonhufvud (1981)). Friedman still made the point (the natural rate of unemployment is the rate observed at the natural rate of interest) in his "Presidential Address" (1969, p.8), but it apparently went unnoticed. Thus, as explained by Dixon (1995), modern macroeconomics is based on ad-hoc assumptions such that the labour market has been "decomposed" from the rest of the system, notably from the capital market.

From this point of view, the NNS is no exception since it is a theory of intertemporal equilibrium with no treatment of constrained allocations at "wrong" prices (see also Van der Ploeg (2005)). This problem surfaces in the so-called forward-looking IS and AS functions of the NNS models. The former is generally obtained upon log-linearizing a standard Euler equation of the representative household (e.g. Woodford (2003, p.246)). The Neo-Wicksellian interpretation of these functions is that actual output is above (below) potential whenever the market real interest rate is below (above) the natural rate. Concomitantly, inflation is above (below) the rate corresponding to potential output. The point is that these IS and AS functions are derived assuming unconstrained transactions whereas, by the very first principles explained above, when the market real interest rate deviates from the natural one households and firms are symmetrically intertemporally constrained on the three markets of capital, labour and goods. The ensuing allocations should lie off the IS and AS schedules. The problem is only addressed by Woodford when he introduces fixed capital in his model (2003, ch. 5) (a feature notably absent from standard NNS models). His task "is to develop a model of optimizing investment demand by suppliers with sticky prices that are demand-constrained as a result" (p. 353). Yet, as explained above, this task is necessary independently of price stickiness and for all agents if the Wicksellian heritage is to be taken seriously.

3.1. The model

To properly formalize saving-investment imbalances, one needs a dynamic model, not in the current sense of the path of continuous intertemporal equilibrium, but in the sense that it should track the behaviour of the system out of equilibrium in the transition from one steady state to another. An example is the following. Let us consider a GE economy characterized by the natural rate of unemployment (NAIRU) $u$ as
determined by a given combination of tastes, technology and the relative value of the real wage rate $w$ with respect to the natural interest rate $r$. Associated with $u$ let there be the inflation rate $\pi$. All the GE variables ($u$, $w$, $r$, $\pi$) are assumed to be constant\(^9\). As dictated by the intertemporal Walras law reminded above, the actual unemployment rate at any time, $u_t$, cannot be equal to $u$ unless the market real interest rate, $i_t - \pi_{t+1}$, is equal to $r$. Also remember that any saving-investment imbalance at time $t$ implies a correspondent labour demand-supply imbalance at time $t+1$. Consequently, unemployment dynamics can be represented by a first-order linear equation like the following

\begin{equation}
    u_{t+1} = u + \rho(u_t - u) + \alpha(i_t - \pi_{t+1} - r)
\end{equation}

where $u_{t+1} \neq u$ as long as $(i_t - \pi_{t+1}) \neq r$, with some degree of persistence $0 < \rho < 1$. This may be called the "cap-lab" (CL) function since it relates the labour to the capital market.

Parellely, the inflation rate at any point in time cannot be at the rate $\pi$ unless contemporaneous unemployment is at the NAIRU, i.e.

\begin{equation}
    \pi_{t+1} = \pi - \beta(u_{t+1} - u)
\end{equation}

where $\beta > 0$ denotes the responsiveness of nominal prices/wages to goods/labour markets deviations from steady state. This is clearly a standard Phillips curve (PC)

Finally, the model is closed by the determination of the expected inflation rate that enters the market real interest rate. As is well known, investors' expectations formation has been matter of never-ending quarrels in the older macroeconomic literature up to the advent of the rational expectations hypothesis. In the context of this model, recourse to the rational expectations hypothesis would imply that agents know the steady-state values of the variables, which in turn depend on the inflation expectation itself. This is the notorious self-referentiality inherent in that hypothesis (see e.g. Evans and Honkapohja (2001)). Our main task here, however, is not to investigate the consequences of different expectational mechanisms. I therefore introduce a close antecedent of the modern rational expectations hypothesis, namely the concept of "normal" inflation rate. The concept of normal value of a variable was largely used as point of reference for expectations by Wicksell, Keynes and pre-Lucasian economists in

\(\text{\(^9\) According to standard DSGE methodology these variables may change over time owing to random shocks to the underlying parameters. This feature is inessential for present purposes.}\)
general. Normality was generally referred to the long-run average value observed for a variable, which is expected to prevail in the future in the states of rest of the system. In our model, this information about inflation is provided by equation (2), so that $\pi$ should be regarded as the "normal" inflation rate:

$$\pi_t^{e+1} = \pi$$

In any case, if the hypothesis of normal inflation elaborated by agents is correct, then $\pi$ should result as the steady-state solution of inflation. If this happens, $\pi$ is also the rational expectation of the inflation rate.

After substituting for inflation expectations, the CL-PC equations form a system of two first-order difference equations with two endogenous variables $[u_t, \pi_t]$, one time-varying exogenous variable, $i_t$, and three constant exogenous $[u, \pi, r]$. The system can conveniently be transformed in terms of two endogenous "gaps" $[\hat{u}_t \equiv u_t - u, \hat{\pi}_t \equiv \pi_t - \pi]$ i.e.

$$u_{t+1} = \rho \hat{u}_t + \alpha(i_t - i)$$
$$\hat{\pi}_{t+1} = -\beta \hat{u}_{t+1}$$

where $i \equiv r + \pi$.

Then it is easily seen that the system achieves the steady state with zero endogenous gaps $[\hat{u}_t = 0, \hat{\pi}_t = 0]$ if and only if

$$i_t - i = 0$$

Expression (6), that I shall call the "interest-rate gap", concerns the nominal interest rate vis-à-vis the "non-accelerating-inflation rate of interest" (NAIRI) that is given by the natural rate plus normal inflation. The expression is exactly equivalent to the difference between the market real interest and the natural rate, but it is more convenient in the present context. The key point is, as required, that unemployment is at the NAIRU only when the nominal interest rate is at the NAIRI. As long as $i_t \neq i$, $\hat{u}_t \neq 0$ in spite of the fact that the real wage rate $w$ is "right" with respect to the natural interest rate $r$.

The main difference with the standard NNS models is that the forward-looking term in their IS function only depends on future interest-rate gaps, whereas equation (4) implies that that term cannot be taken as

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10 A more subtle interpretation of the normal value of a variable (certainly one to which Keynes paid great attention) is that it does not necessarily come as a structural information, but it rather consists of a general belief or "convention" of the agents. We shall return to this distinction later.

11 All analytical proffs are in Appendix
independent of the current interest-rate gap\textsuperscript{12}. This is not a trivial difference, as we shall see. Another point that deserves a discussion is the interpretation of the natural interest rate. The "New Synthetics", according to modern methodology, identify the latter as a GE variable determined by "deep parameters" (households' intertemporal marginal rate of substitution, or, if physical capital is present, the marginal return rate to capital – e.g. Woodford (2003, chs. 4 and 5)). In so doing, they can apparently rely on Wicksell's idea of the natural rate being determined by the forces of "productivity and thrift" and, \textit{in equilibrium}, being coincident with the marginal product of capital (1898b, p.82). However, a subtle point of difference is that Wicksell regarded the natural rate, so to speak, as a variable "hidden from view". Like the equilibrium price in any market, the natural interest rate, too, may be unknown to traders in advance: it is the task of the "invisible hand" to drive the traders to discover it by reacting to out-of-equilibrium market signals\textsuperscript{13}. Indeed, our CL-PC equations can simply be understood as a means to check whether the system "finds" a particular value of the real interest rate that is consistent with the GE steady state.

\subsection*{3.2. Simulations}

As regards the system's out-of-equilibrium dynamics, the CL-PC model is able to reproduce the effects of various types of shocks, namely \textit{real shocks to the NAIRI} \(i\) (i.e. shocks to the natural rate \(r\)), \textit{nominal shocks to the NAIRI} \(i.e.\) shocks to the normal inflation rate \(\pi\), \textit{nominal shocks to the market interest rate} \(i_t\) and \textit{real shocks to the NAIRU} \(u\).

In the following I will present simulations of the model. All the variables have been treated as index numbers, and their initial steady state values normalized to 100. The parameters have been set to the following values:

\[ \rho = 0.1, \alpha = 0.3, \beta = 0.5 \]

\textsuperscript{12} For instance, the IS function derived by Woodford (2003, ch.4) is

\[ x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r) \]

where \(x\) is a measure of output gap (to which there corresponds a measure of unemployment gap). By iteration,

\[ x_t = E_t x_{t+s+1} - \sigma \sum_{s=0}^{\infty} (i_{t+s} - E_t \pi_{t+s+1} - r), \quad s = 0, 1, \ldots \]

\textsuperscript{13} I thank Axel Leijonhufvud for having drawn my attention to this interpretation of Wicksell, and of my own model.
Of course, these values have no particular empirical meaning. They portray a system where persistence is low, interest rate gaps have a moderate effect on the unemployment gap, and these have a smooth effect on the inflation gap. These parameters also ensure that, as long as the interest-rate gap is nil, the system is stable. Thus, we shall be able to understand under what conditions different interest-rate adjustment mechanisms contribute to stability or not.

In the first place, let us examine the dynamic response of the system to a typical Wicksellian shock, that is a nominal shock to the market interest rate. Let us suppose that the nominal interest rate rises by 10 basis points permanently.

**Figure 1. Permanent positive nominal shock to the market interest rate. Time diagram**

![Diagram showing the response of unemployment and inflation to a permanent positive nominal shock to the market interest rate.](image-url)
The shock implies that the nominal interest rate exceeds the NAIRI (the market real interest rate exceeds the natural rate).

As a consequence, excess saving gives rise to a combination of excess unemployment and deflation.

As long as the interest rate gap is not closed, the system settles down in a state of excess unemployment and negative inflation gap.

The Phillips curve is negatively sloped and the observations are due to transition to the new unemployment-inflation steady-state gaps.

### 3.3. Discussion

In this model, the steady-state inflation gap is consistent with Wicksell’s idea of the "cumulative process". Suppose, as Wicksell did, that the initial steady state is one with constant price level. Then, our result means that the price level would indefinitely fall at a constant rate (Wicksell (1898b, pp. 77-78)). Wicksell, correctly, considered these price changes a major disequilibrium phenomenon which should carefully be understood and curbed, though they may occur in perfectly competitive goods and labour markets (in which case the NAIRU $u$ would simply be zero). On the other hand, he did not pay sufficient attention to the real side of the disequilibrium cumulative process.
Thus, the steady-state unemployment gap may be regarded as a characterization of Keynes’s concept of “involuntary unemployment”, with a caveat to be discussed below. Given the "wrong" market real interest rate, not all workers ready to work at the GE real wage rate will ever be employed. Since no structural parameter has changed that justifies a change in the real wage rate, the unemployment gap is entirely due to the interest-rate gap. Also, the extent of the unemployment gap is independent of the responsiveness of inflation to the unemployment gap itself (parameter $\beta$ in equation (2)), that is, it is independent of the degree of price flexibility. As already anticipated in the theoretical discussion, the only reason why this feature may be relevant is through its interplay with the interest-rate gap via expected inflation. Having assumed a constant normal inflation rate to which the system is expected to return, and hence a constant interest-rate gap, the actual short-run changes in the price level are ininfluential.

The role of expected inflation leads us to the caveat concerning "involuntary unemployment". The point, as seen above, is that any permanent interest-rate gap determines a steady state of the system that falsifies the return-to-normality hypothesis of the inflation rate. Hence the unemployment gap is associated with less-than-expected inflation, a well-known argument against the consistency of "involuntary unemployment" as a steady-state. As an alternative, let us consider "short-run" rational expectations, that is the correct forecast of one-period-ahead inflation during the adjustment process. Hence replace equation (3) with

\begin{equation}
\pi_{t+1} = \pi_{t+1}
\end{equation}

Consequently, equation (4) should also be replaced by

\begin{equation}
\hat{u}_{t+1} = \rho' \hat{u}_t + \alpha'(i_t - i)
\end{equation}

where \( \rho' = \frac{\rho}{1 - \alpha' \beta} \), \( \alpha' = \frac{\alpha}{1 - \alpha \beta} \).

It is clear that unemployment dynamics now comes to depend on the parameter $\beta$. However, as long as $\alpha \beta < 1$, and $\rho' < 1$, $\alpha' < 1$, the system does not change its qualitative properties, whereas it tends towards instability as $\beta$ increases – a well-known argument by Keynes (1936, ch. 19). Quantitatively, it can be noted that, if $\alpha \beta < 1$, then $\rho' > \rho$ and $\alpha' > \alpha$. This means that not only do short-run forward-looking investors not prevent the system from settling down at non-zero unemployment and inflation gaps, they also amplify these gaps. Consider again an increase in the nominal interest rate as before. Now investors anticipate the ensuing fall in the
inflation rate. As a consequence, the gap of the market real interest rate relative to the natural rate is amplified and so are the unemployment and inflation gaps along the adjustment path.

So far the nominal interest rate has been treated as an exogenous variable. Our next step will be to close the model with an adjustment equation of the nominal interest rate $i_t$ that endogenizes the dynamics of the interest rate gap after an initial shock.

4. Endogenizing the nominal interest rate

For the sake of comparison, I first consider three different specifications inspired by alternative theories of the interest rate: 1) a Wicksellian bank mechanism, 2) a "dynamic" Keynesian LM equation, 3) a "speculative" LM equation. All three specifications relate to market forces driving the nominal interest rate, with no explicit role for monetary policy

4.1. The Wicksellian bank mechanism.

The well-known Wicksellian idea is that the out-of-equilibrium nominal interest rate is procyclical with economic activity and the price level (e.g. (1901, Bk. II), (1898b)). The reason is that banks raise or lower their nominal rate to the extent that the price level increases above or decreases below its "normal level". This process is driven by the need of banks to keep their loans balanced with real reserves during the expansion (contraction) of the demand for funds and of the price level. As explained in section 2, this mechanism hinges on a limited informational requirement, in that banks need not know what the natural rate is at each point in time, which is consistent with the idea that the nominal interest rate may assume wrong values\(^{14}\).

Let $\pi$ be the rate of inflation that also banks take as normal. Then the nominal interest rate goes on rising (falling) as long as inflation accelerates (decelerates) with respect to the rate $\pi$, that is to say

\[^{14}\text{Apparently, the Fisher equation provides an analogous mechanism that may relate the market interest rate to inflation, but there is an important underlying difference, which is immediately brought to light by this exercise. The Fisher equation holds in steady state as shown by expression (6), but we cannot use it consistently as an out-of-equilibrium mechanism because it would imply that the market rate instantly adjusts itself to the NAIRI at each point in time. Therefore, there would never be any interest rate gap!}\]
\[ i_t = i_{t-1} + \gamma(\pi_t - \pi) \]

On adding this equation to the CL-PC system, we obtain three first-order difference equations in three endogenous variables \([u_t, \pi_t, i_t]\) and three constant exogenous \([u, \pi, r]\). After rearranging in terms of the endogenous gaps \([\hat{u}_t \equiv u_t - u, \hat{\pi}_t \equiv \pi_t - \pi, \hat{i}_t \equiv i_t - i]\), it can be seen that the system admits of a zero-gaps steady-state.

As to convergence and stability of the system, it is interesting to restrict our analysis to monotonic convergence conditions. For an interest rate mechanism associated with oscillations of the system, even though convergent, would neither be interesting nor recommendable. In the present case, for any \(\gamma > 0\), the following single condition is sufficient

\[ \gamma < \frac{(1 - \rho^{1/2})^2}{\alpha \beta} \]

Hence we can conclude that the Wicksellian bank mechanism is stabilizing: as the nominal interest rate converges to the NAIRI, unemployment converges to the NAIRU and the return-to-normality hypothesis of the inflation rate is fulfilled. Notably, the nominal interest rate converges to the NAIRI despite this variable is not made explicit in the interest-rate equation. Yet stabilization occurs only within a given range of the sensitivity of the nominal interest rate \(\gamma\) to inflation dynamics (less than 3.12 with our parameters). The economic meaning of this condition can be understood by noting that \(\gamma \alpha \beta\) measures how one point of interest-rate gap that triggers \(\alpha\) points of unemployment gap is self-corrected through the response \(\gamma\) of the nominal interest rate to the \(\beta\) points of inflation gap generated by the unemployment gap. As is intuitive, a stabilizing adjustment mechanism requires that \(\gamma\) should be smaller, the larger are \(\alpha\) and \(\beta\). As \(\gamma\) increases, the system first takes an oscillatory path and then becomes unstable.

**Simulations**

A) I first report the result of the same previous nominal shock of +10 basis points to the nominal interest rate. The parameter \(\gamma\) has been set equal to 0.8, which satisfies condition (10).
Figure 3. Positive nominal shock to the nominal interest rate. Time diagram with the Wicksellian bank mechanism

Figure 4. Positive nominal shock to the nominal interest rate. Phillips curve with the Wicksellian bank mechanism
• On impact unemployment rises and inflation falls  
• As inflation falls below the normal level, the nominal interest rate is reduced until the initial interest-rate gap is closed  
• *The shock has only transitory effects on all the real and nominal variables owing to the progressive absorption of interest rate gap* 
• The Phillips curve shows a negatively sloped plot; observations are due to transitory dynamics.

B) I now examine the symmetric case of a negative *real* shock to the NAIRI, that is a fall in the natural interest rate $r$ by 10 basis points (new NAIRI = 90). This case is symmetric to the previous one because it implies that initially the nominal interest rate lags behind and hence the system experiences a positive interest rate gap (e.g. Wicksell (1898b)). This case is also central to the Keyensian literature from the origin (the fall in the marginal efficiency of capital in the *General Theory*) up to the NNS (see e.g. Woodford (2003)).

**Figure 5. Permanent real shock to the NAIRI. Time diagram with the Wicksellian bank mechanism**

• On impact, the nominal interest rate lags behind *vis-à-vis* the fall in the NAIRI: a positive interest-rate gap opens up
Subsequently, unemployment starts raising above the NAIRU, with inflation falling below the normal rate.

The nominal interest rate also starts falling by keeping pace with deflation.

*The shock has only transitory effects on unemployment and inflation, owing to the progressive absorption of the interest-rate gap, and permanent effects on the nominal interest rate, owing to the fall in the natural rate.*

The Phillips curve (not reported) records a negatively sloped plot analogous to simulation A.

**Discussion**

The Wicksellian bank mechanism exemplifies the class of *endogenous* interest-rate processes established by Woodford (2003, ch. 4) that progressively absorb the initial interest-rate gap, whether this is due to a real or a nominal shock. In each instance examined, the interest-rate adjustment is consistent with the economy’s convergence to the steady state and supports a determinate rational-expectations equilibrium as claimed by Woodford. It should be noted, however, that the process devised by Wicksell is a *market, spontaneous* mechanism with no *explicit* role of the central bank in the determination of the interest rate. To this point we shall return shortly.

In the present model, the rational-expectations equilibrium supported by the system is the return-to-normality hypothesis. This result hinges on the generalized belief in the normal inflation rate \( \pi \). To be precise, what the model actually says is that *any belief concerning the normal inflation rate consistently held by all agents is self-fulfilling*. As is well known, Wicksell and his followers were aware of, and worried about, the indeterminacy of the inflation rate over pure credit cycles (see e.g. Boianovsky and Trautwein (2004, 2006)). For instance, this model vindicates their concern that extrapolative expectations by investors \( (\pi_{t+1}^e \text{ replaced by a function of } \pi_t) \) would lead to divergent cumulative processes (not reported here). And this is, of course, an important message for monetary policy. If a spontaneous bank market mechanism like (9) may be relied upon in order to correct deviations from the natural interest rate, a crucial role for the central bank nonetheless remains as "manager of expectations" (Woodford (2003, pp. 15-17)). In modern parlance, this role would consist in anchoring the public’s
expectations to a consistently (credibly) realized normal rate of inflation. Hence Woodford is right when he stresses the remarkable modernity of this Wicksellian view of central banking and its consistency with the modern theory and practice of monetary policy rules.

Indeed, it is almost trivial to observe that an interest-rate mechanism like (9) is substantially similar to a rule of inflation targeting with interest-rate smoothing (Svensson (1997)). Thus, one may interpret (9) as the reduced form of a set of inter-bank relationships whereby the central bank drives the interest rate on loans, with the anchor of expected inflation being explicitly set by the central bank as a target.

A final consideration concerns the Keynesian issue of "involuntary unemployment". As is clear, under the Wicksellian bank mechanism excess unemployment arises over the cycle but not in the steady state. Hence, explanations of involuntary unemployment as a steady-state phenomenon should look for other mechanisms that prevent the interest rate gap from being closed (see also Leijonhufvud (1981)). The next candidate is of course the Keynesian monetary theory of the interest rate.

4.2 Dynamic LM

These exercises have the instructive by-product of making it immediately clear that the standard specification of the LM equation cannot be used to address the problem at hand, which is intrinsically dynamic. Thus the numerous complaints about the "static" nature of the LM and its misconception of genuinely Keynesian economics seem vindicated (Leijonhufvud (1983)).

I have thus devised a "dynamic LM" equation for the nominal interest rate in the following way. Let us start from the textbook LM function which represents the interest rate as a function increasing in current real income and decreasing in real money supply\(^{15}\). If \(\mu_y\) and \(\mu_i\) are the income and interest-rate elasticities of money demand, then \(1/\mu_i = \delta\) and \(\mu_y/\mu_i = \mu_y\delta\) are the elasticities of the interest rate relative to real money supply and real

\(^{15}\) The typical LM function is obtained by starting from a log-linear money demand function,

\[ m^d_t = \mu_y y_t - \mu_i i_t \]

Equating money demand to real money supply, \(m_t - p_t\), the equilibrium interest rate is

\[ i_t = (\mu_y/\mu_i)y_t - (1/\mu_i)(m_t - p_t). \]
income, respectively. This theory implies that the interest rate is constant over time as long as real income and real money supply are constant. Assuming a log-linear relationship between output (income) and unemployment via production function, a simple dynamic equation consistent with this theory is the following:

\[ i_t = i_{t-1} - \mu_y \delta (u_t - u_{t-1}) - \delta (\hat{m}_t - \pi_t) \]

where \( \hat{m}_t \) is the growth rate of money supply.

Also this mechanism has a very limited informational requirement, being entirely based on step-by-step adjustment to current observable variables with no need for public information on the natural interest rate. There are two main differences with respect to the Wicksellian bank mechanism, however. One is the sensitivity to unemployment dynamics, the other is the dependence on an exogenous dynamic variable – money supply.

Adding equation (11) to the system CL-PC system and rearranging, the steady-state values of the endogenous gaps are still zero provided that \( \hat{m}_t \) is constant and equal to the normal inflation rate \( \pi \). Given \( \hat{m}_t = \pi \), we now have the following condition for monotonic convergence and stability:

\[ \delta < \rho/\alpha \mu_y \]

The (striking?) economic message is that the "monetary determination" of the interest rate by way of a plain dynamic LM function is still compatible with overall system stability and convergence to the "natural" variables. The only relevant point is that the properties of the system now crucially hinge on the relationship between the parameters that relate the nominal interest rate to both inflation and unemployment. In particular, stability implies an inverse relationship between the two. On the other hand, the smaller is \( \delta \), the smoother is the interest rate dynamics and the longer is the whole adjustment process.

**Simulations**

A) In the first place, let us focus on the case of a permanent real shock to the NAIRI, which is most typical of the Keynesian literature. As in the Wicksellian case, the natural interest rate falls by –10 basis points (new NAIRI = 90). The parameter \( \mu_y \) has been set equal to 0.5. For the sake of comparison, the value of parameter \( \delta \) has been set equal to 0.5, like the analogous parameter \( \gamma \) in the Wicksellian simulation, and in line with condition (12), which in our case is 0.67.
The shock has the same qualitative effects as in the Wicksellian case. Excess unemployment is reabsorbed along with the positive interest rate gap.

After the positive interest rate gap at time 0, unemployment rises and deflation is triggered.

The income and real-balance effects work: they progressively reduce the nominal interest rate and close the interest rate gap.

The Phillips curve would display the same plot as the analogous Wicksellian case (see Figure 4).

B) Let us now examine another typical Keynesian exercise, that is a temporary positive shock to the money growth rate, which increases by 10 basis points at time 0 and then returns to the normal level. Clearly, this is analogous to a negative nominal shock to the market interest rate.
The shock has transitory effect on all variables

- On impact, the nominal interest rate falls below the NAIRI and cuts unemployment below the NAIRU
- Thereafter, the increase in inflation reduces real money balances and drives the interest rate to its initial steady-state value
- The Phillips curve (not reported here) would still be negatively sloped with observation being given by transitory dynamics.

Discussion

The most important lesson to be drawn from this exercise is that even a Keynesian LM interest-rate equation closely reproduces the results of the Wicksellian one. From the Keynesian (Old Synthesis) point of view, three are the implications. The first is that the Keynesian theory of the interest rate does not seem, per se, sufficient to explain a steady state with involuntary unemployment. The economy, as in the Wicksellian view, seems to be endowed with reliable self-stabilizing mechanisms (in particular the real balance effect on money demand) in the cases of both real and nominal business cycles. The second is that, apart from accelerating and smoothing the adjustment process, little scope is left for monetary policy. Its most important role is instead more Friedmanite than Keynesian. As stressed in
the discussion of the Wicksellian case, the self-stabilizing property of this economy still relies on the agents’ common belief in the normal inflation rate $\pi$, which is in fact realized in the steady state. In this case, however, this belief is not arbitrary but can (or should) be anchored to a given state variable, namely the money growth rate. In fact, the model shows that a) a temporary deviation of the money growth rate from the expected normal inflation rate has only temporary real effects, b) a permanent deviation would have permanent real effects but with a permanent inflation gap that would falsify the return-to-normality expectation, c) if agents adapt their expected inflation rate to the new money growth rate, then the system goes back to the zero-gaps steady state, i.e. the real effects of the change in the money growth rate vanish. Overall, these implications amount to the Monetarist interpretation of the Old Synthesis (see also Leijonhufvud (1981)).

4.3. Speculative LM.

The last alternative determination of the nominal interest rate to be examined follows from one among the many criticisms raised against the textbook LM version of Keynes’s theory of the interest rate. The thrust of this criticism is that one major element in that theory, the "speculative motive" of the demand for money, has gone completely astray (Leijonhufvud (1981)). A truly "speculative" component of money demand should be related to expected movements of the interest rate relative to its future value, say $i^e$. Speculators substitute bonds for money whenever they expect capital gains, i.e. a rise in the bonds price or else a fall in the market interest rate. Therefore, this component should enter the usual representation of money demand as a negative function of $(i_t - i^e)$ (Leijonhufvud (1981), p.146). The dynamic LM should therefore be rewritten as follows

$$i_t = i^e + \mu \gamma \delta (y_t - y_{t-1}) - \delta (\hat{m}_t - \pi_t)$$

This specification implies that as long as real income and real money supply are constant, speculation keeps the market interest rate aligned with $i^e$.

For brevity I do not report here the analytical results of the new model. Attention should be drawn to the point that equation (13) reintroduces an exogenous constant, $i^e$, into the model. The consequence is that now the zero-gaps steady state can only be attained if $i^e = i$. That is to say, if the speculators' expected interest rate is the NAIRI, then the market interest rate does convergence to the NAIRI, otherwise it does not. In the
former case, the convergence and stability conditions are slightly different than in the plain LM case. In particular, as can been seen from the simulation below, the same values of the parameters now yield oscillatory convergence. But this is not the main point, which is instead that now the determination of the nominal interest rate has a crucial informational requirement, that is, \( i^e \). However, not only Keynes (1937), but also Wicksell (1898b), were well aware that the natural interest rate may be volatile, it is hardly observable and measurable, and that timely and exact information about its value at specific points in time may be chimerical outside firms, to say the least.

It is therefore of particular importance and interest examine the system’s behaviour in two situations vis-à-vis a permanent change in the natural rate, one where speculators have this information and one where they do not.

**Simulations**

Let us start from a permanent real shock to the NAIRI such that the natural interest rate falls by 10 basis points (new NAIRI = 90). The parameters are the same as in the previous LM model.

A) Let us first consider the full information case in which speculators exactly anticipate the new NAIRI, i.e. \( i^e = i = 90 \).

**Figure 8. Permanent real shock to the NAIRI. Time diagram with speculative LM and full information**
• The shock has the same qualitative effect as the plain dynamic LM.
• The initial impact on the interest rate is larger since it embodies the lower NAIRI, which is however overshot owing to the concurrent reduction in money demand exerted by higher unemployment and lower inflation
• Initial overshoooting of the market interest rate generates oscillatory dynamics.

B) We now examine the same previous shock under limited information, where speculators do not revise their expected value of the NAIRI, i.e. $i^e = 100$.

Figure 9. Permanent real shock to the NAIRI. Time diagram with speculative LM and limited information

- The shock has permanent effects on both real and nominal variables. In the steady state, unemployment is above the NAIRU, the nominal interest rate is above the NAIRI, and inflation is below the normal rate.
- The Phillips diagram (not reported here) would show a negatively sloped schedule with observations given by permanent transition to the new steady state (see e.g. Figure 2)
Discussion

The scenario under limited information resembles the initial one with exogenous nominal interest rate (section 3), and, again, it seems to have genuine Keynesian features, in that "involuntary unemployment" emerges because the speculative demand for money prevents the market interest rate from falling enough to offset the fall in the natural rate. The fundamental cause is that speculators do not adjust their expected rate to the lower NAIRI. There are however several problems hidden behind this result:

- the exercise has been run holding the expected inflation rate $\pi$ constant at 100: yet this expectation turns out to be wrong; the steady state inflation will be lower (thus the new steady-state NAIRI is even lower than after the initial shock since both $r$ and $\pi$ have fallen)
- in the new steady state the inflation rate is lower than the money growth rate (held constant at 100); hence real money balances keep on growing and create excess purchasing power in the hands of speculators
- on the other hand, the market interest rate stabilizes at a value lower than $r^e = 100$ expected by speculators, who should therefore keep on anticipating capital losses in the bonds market, which prevent them from buying bonds.

It therefore seems that the steadiness of the "wrong" nominal interest rate is due not to the absence of forces of change, but to two countervailing out-of-equilibrium forces. It is tempting to see here a possible manifestation of the liquidity trap (clearly any further increase in the money growth rate would be useless). If this is the case, it seems necessary to conclude that the liquidity trap cannot be regarded as an extreme case in the Keynesian pathology but is indeed the Keynesian pathology! Are therefore Pigou and Modigliani vindicated? Not exactly. A methodological point made by Leijonhufvud in the "Wicksell Connection" (1981) applies here, namely that the pathological states of the system are not due to structural parameters but to particular combinations of events and the way in which they are processed by markets. In fact, the pathology we have found is not related to anomalous liquidity preference (the relevant parameter is always the same) but to an informational/expectational error. The implications concerning the relevance of the problem are quite different.

On the one hand, being fraught with expectational errors this scenario can hardly be considered a genuine steady state. This finding
probably frustrates the Old Keynesians' search for "involuntary unemployment equilibria". On the other hand, it is also challenging in that it points out at least one case in which, in a well-specified sense, a purely market-driven interest rate may put the system on the wrong track. Moreover, it is difficult to see where the system can be driven from here, since the corrections of the underlying errors may prove far from smooth and painless.

The previous conclusion prompts further thoughts on monetary policy as a visible hand possibly keeping the interest rate on the right track. A natural candidate is the most popular policy rule on the menu of modern central banks: the Taylor rule. This rule has a variety of specifications which can be summarized as follows:

\[
i_t = i^* + \phi(y_t - y) + \theta(\pi - \pi)\]

where \(i^*\) is the benchmark nominal rate, and \(\pi\) is the informational inflation rate (the inflation rate used to assess the cyclical position of the economy).

In empirical versions, the benchmark rate \(i^*\) is often replaced by the so-called "interest-smoothing" mechanism, i.e. a gradual adjustment of the interest rate from the previous period, and current inflation is used as informational input. Thus, if we set \(i^* = i_{t-1}\) and \(\pi = \pi_t\) we obtain an "adaptive Taylor rule".

Remarkably, this specification is analytically equivalent to our dynamic LM, provided that a) there exists a log-linear relationship between output and unemployment, and b) the money growth rate is replaced by the target inflation rate. The former is a sheer technical detail which is easily met, the latter reflects the shift from the Friedmanite view of monetary policy conduct (based on the control of the money growth rate, with the nominal interest rate as an endogenous variable) to the NNS view (based on the control of the nominal interest rate, with the money growth rate as the endogenous variable). For unclear reasons, the belief is growing that the straightforward use of the Taylor rule instead of the discredited LM equation in standard macroeconomic models is a major improvement (especially for teaching purposes: e.g. Taylor (2000), Carlin and Soskice (2004)). Yet once the LM function is properly dynamized, it indicates how the central bank should gear the money growth rate in order to control the capital market interest rate, \(vis-à-vis\) the current inflation rate and a real indicator of the business cycle. The Taylor rule assumes as a short-cut that
the central bank can set this rate directly. In a more refined framework, the
dynamic LM and the Taylor rule can only be complementary, since it is hard
to see how the central bank can control anything if it is not consistent with
money demand and supply (flow) equilibrium. In fact, the true instrument
that the central bank can control directly is the discount rate, which is in its
turn a means to regulate credit creation and hence the money growth rate.
If the Taylor rule, as in its empirical applications, is taken to represent how
the central bank wishes to manage the discount rate, then, assuming a
stable relationship between the discount and money growth, the dynamic
LM equation yields the changes in the interest rate consistent with the
equilibrium of money demand and supply. Having the same properties as
our dynamic LM function, we can conclude that the Taylor rule belongs to
the class of stabilizing interest-rate mechanisms as argued by Woodford
(2003, ch.4) – with a few caveats to be discussed below.

As to the exhibited Wicksellian pedigree of the Taylor rule, it is
indeed easy to see that it consists of the Wicksellian bank mechanism plus
the sensitivity of the interest rate to output gaps. The inclusion of the latter,
as can be inferred from previous analysis of the dynamic LM, may only
change the speed and path of the dynamics of the endogenous variables
unless the relevant parameter is very high, in which case it may destabilize
the system. However, a major point of which the Taylor-rule scholars seem
unaware is that the choice of the two parameters in the rule has to take into
account the system’s stability conditions. The reason why the two
parameters should be inversely correlated as dictated by condition (12) is
easily understood upon considering the cases in which the output and
inflation gaps are positively correlated. Then the mechanical application of
the Taylor rule would imply that the interest rate reacts twice to the same
signal. Hence the system’s stability requires that this compound response be
limited upwards. In other words these parameters cannot simply reflect the
central banker’s preferences in an unconstrained optimization exercise.

A further implication is that the so-called "Taylor principle", that is
the requirement that the inflation-gap parameter $\theta$ be greater than 1
(Woodford (2001)), is neither necessary nor sufficient. For particular
combinations of very low persistence ($\rho$) and/or very high elasticity ($\alpha$) of
output gaps with respect to interest-rate gaps, $\theta > 1$ might even turn out to
be destabilizing. On the other hand, once the relevant stability condition is
verified, $\theta < 1$ may well be sufficient.
Specific consideration finally deserves the prescription that the Taylor rule should be pegged to the natural rate of interest (Woodford (2003, ch. 4)), that is, \( i^* = i \) in our formulation of equation (14). This is analytically equivalent to our speculative LM function, when speculators know the right NAIRI, with the exogenous money growth rate \( \hat{m} \) replaced by the target inflation rate \( \pi \). Indeed, the same conclusion applies: namely, that targeting the NAIRI directly may be dangerous when it is not known with certainty. In a recent study published by the ECB, one reads that from the empirical point of view, the "natural" real interest rate is unobservable. The estimation of the natural real interest rate is not straightforward and is associated with a very high degree of uncertainty (Garnier and Wihelmsen (2005), p.6).

If the central bank has complete and immediate information about the NAIRI, then it can and should immediately adjust the nominal interest rate to offset any change in the NAIRI as it arises. If the central bank does not have this information, and it happens to peg the nominal interest rate to the wrong NAIRI, then the Taylor rule would drive the system towards a permanent output gap, like the speculative LM function with limited information.

5. Conclusions

Let me summarize the main findings of this renewed exploration of the connections between Wicksell, Keynes and modern macroeconomics.

First, the main distinction of the original Wicksellian theory with respect to the DSGE framework of the NNS can be seen in the focus on disequilibrium business cycles. This type of cycles are driven by "interest rate gaps" between the nominal interest rate and the NAIRI and by saving-investment imbalances. Nominal wage-price stickiness is not the only problem, wage-price flexibility is not the only solution (in fact, Wicksell’s concern with this type of cycles was not related to any role of wage-price stickiness).

Second, these cycles are benign as long as the system embodies an endogenous mechanism that drives the nominal interest rate to close the gaps with the NAIRI, with two anchors in which agents have reason to believe: a "nominal anchor", a normal inflation rate, and a "real anchor", a normal real interest rate on capital. Under these conditions, the economic
system is probably more robust than the Old Keynesians (and Keynes?) believe(d), and the mere existence of the interest elasticity of money demand is not an impediment. Nonetheless, this class of cycles remains relevant to the extent that interest rate gaps are frequent, substantial and persistent. Even when the long-run dynamics is benign, frequency, amplitude and persistence of these cycles may make them problematic enough in the short to medium run. A minimal list of requirements for relevance (and concern) is that the NAIRI should be volatile and that it should not be easily transmitted to the capital market; since the NAIRI consists of the natural rate plus normal inflation, these requirements should apply to both components, or at least to one.

Third, if informational problems with a volatile NAIRI are the crux, interest-rate mechanisms relying upon timely and precise knowledge of the NAIRI are inapplicable in that they simply rule the problem away; simulations have also shown that these mechanisms are destabilizing if they embody the wrong NAIRI. Thus, unless we can be highly confident that central banks are better (perfectly) informed than the market about the natural rate of interest, "adaptive" rules, using step-by-step adjustments of the interest rate vis-à-vis observable conditions in the economy are preferable in that they produce adjustment paths which are generally slower, but safer.

Having said this, the model employed in this paper is open to an alternative interpretation of the robustness of its stabilizing mechanisms: these mechanisms drive the system back to any value of the NAIRI and the NAIRU in which the agents (the central bank) consistently believe, that is to say, these beliefs are self-fulfilling. The next question is therefore whether there exists, or how it can be created, a deeper anchorage of these beliefs in a normative sense.

Further "complications" that may arise from other perspectives and that are worth exploring are the following:

- Keynes (1937), Lindahl (1939), New Keynesians à la Greenwald and Stiglitz (1993), and Woodford on passing (2003, ch. 5), would add that the deviations of the market real interest rate from the natural rate do not leave the capital stock unaffected (that is a straightforward implication of the fact that saving-investment imbalances impinge upon aggregate demand, employment and output). If the capital stock changes over the cycle, then the real return to capital also changes. Thus, as
Woodford recognizes, we (or the agents in the economy) off the steady state face three interest rates: the market real interest, the actual real return to capital, and the natural interest rate. Yet all this blurs the notion of a given natural rate of interest independent of the cycle to which the economy should return, and we are led back to the question of the normative anchorage of the belief in a particular natural rate.

- A somewhat more radical perspective would add behavioural finance as a repertoire of causes for the mispricing of firms' investments and consequent misbeliefs in the natural interest rate.
- Neo-Hicksians (e.g. Amendola and Gaffard (1998)) stress that "technological shocks" (possibly underlying the volatility of the NAIRI) are as such non existent (e.g. they remain ideas in the mind of entrepreneurs) until they are "validated" by financial means; in this perspective, changes in the NAIRI are not independent of monetary policy and the market interest rate.

References


Appendix

This Appendix contains the analytical solutions of the various systems of linear dynamic equations used in the main text. Let us start with the CL-PC model in "gaps" form, which is reproduced here for convenience:

(A1) \( \hat{u}_{t+1} = \rho \hat{u}_t + \alpha (i_t - \bar{i}) \)

(A2) \( \hat{\pi}_{t+1} = -\beta \hat{u}_{t+1} \)

where \( \hat{u}_t = u_t - \bar{u} \), \( \hat{\pi}_t = \pi_t - \pi \).

The model is closed by an additional linear dynamic equation for the nominal interest rate \( i_t \). Presented below is the system’s solution for each specification of this equation treated in the text.

A.1. The Wicksellian bank mechanism

In this paragraph I first introduce the general solution method used in all subsequent cases. Let us consider the following equation for the nominal interest rate:

(A3) \( i_t = i_{t-1} + \gamma (\pi_t - \bar{\pi}) \)

Upon subtracting \( i \) to both sides of the equation, and taking one period ahead, this can be re-written as

\( \hat{i}_{t+1} = \hat{i}_t + \gamma \hat{\pi}_{t+1} \)

Addition of this interest-rate equation to system (A1)-(A2) and algebraic substitutions yield a two-equations homogeneous system in the "gaps" \( g'_{t} = [\hat{u}_t, \hat{i}_t] \). In matrix form:

(A4) \( g_{t+1} = Ag_t \)

where \( A \) is the following coefficient matrix:

\[
A = \begin{bmatrix}
\rho & -\alpha \\
\gamma \beta \rho & 1 - \gamma \beta \alpha 
\end{bmatrix}
\]

Given the non-zero matrix \((I - A)^{-1}\), system (A4) has steady-state solutions \( g = 0 \), i.e. \( \bar{u} = u \), \( \bar{i} = i \), which imply, \( \bar{\pi} = \pi \).

Matrix \( A \) can then be used to examine the convergence and stability properties of the system. For economic reasons, we want to study conditions of monotonic stability. This requires the characteristic equation of matrix \( A \) to have two positive real roots smaller than 1. The characteristic equation is given by

\[
\text{det}(A - kI) = 0\\
k^2 - (1 + \rho - \gamma \beta \alpha)k + \rho = 0
\]
Let us refer to the canonical quadratic equation
\[ k^2 + bk + c = 0 \]
The condition for real roots is
\[ b^2 - 4c \geq 0 \]
which is satisfied for
\[ \gamma < \frac{(1 - \rho^{1/2})^2}{\alpha \beta} \]
Since (A6) implies \( b < 0 \), given that \( c > 0 \), the roots are positive. Moreover, any \( \gamma > 0 \) ensures that the largest root is smaller than 1 (\( b > - (1 + c) \)). Therefore, (A6) is necessary and sufficient for monotonic convergence and stability of system (A4).

A.2. Dynamic LM

Let us now consider the following interest-rate equation
\[ i_t = \tilde{r} + \mu y \delta (y_t - y_{t-1}) - \delta (\hat{m}_t - \pi_t) \]
where \( \tilde{r} \) admits of two specifications: a) \( i_{t-1} \), b) \( i^e \)

Under specification a), addition of equation (A7) to system (A1)-(A2) introduces one time-varying exogenous variable, \( \hat{m}_t \), so that the reduced-form system in "gaps" is
\[ g_{t+1} = Ag_t + b(\hat{m}_t - \pi) \]
Therefore, a necessary condition for the steady-state solution \( g = 0 \) is \( \hat{m}_t = \pi \).

The coefficient matrix \( A \) is
\[ A = \begin{bmatrix} \rho & -\alpha \\ \delta (\beta \rho - (1 - \rho) \mu_y) & 1 - \alpha \delta (\beta + \mu_y) \end{bmatrix} \]
with characteristic equation
\[ k^2 - (1 + \rho - \alpha \delta (\beta + \mu_y)) k + (\rho - \alpha \delta \mu_y) = 0 \]
The condition for real roots is
\[ (1 + \rho - \alpha \delta (\beta + \mu_y))^2 - 4(\rho - \alpha \delta \mu_y) \geq 0 \]
and, given \( \rho < 1 \), it is always satisfied for any positive \( \delta, \mu_y \). Then the system has two positive roots smaller than 1 provided that
\[ 1 + \rho - \alpha \delta (\beta + \mu_y) > 0 \]
\[ \rho - \alpha \delta \mu_y > 0 \]
\[ 1 + \rho - \alpha \delta (\beta + \mu_y) < 1 + \rho - \alpha \delta \mu_y \]
Given that \( \beta > 0 \), the third condition is always verified. Since the second condition implies the first, it is sufficient that
\[ \delta \leq \rho / \alpha \mu_y \]
Specification b), $\tilde{I} = i^e$, introduces the additional exogenous fixed variable $i^s$, so that,

\begin{equation}
(A12) \quad g_{t+1} = A g_t + B x_t
\end{equation}

with $x'_t = [(\hat{m}_t - \pi), (i^e - i)]$. Therefore we now have two necessary conditions for the steady-state solution $\bar{g} = 0$, namely $\hat{m}_t = \pi$, $i^e = i$. Given these conditions, the stability and convergence conditions remain unchanged.